

Arithmetic and Geometric Sequences as Functions

Grade 9 – Algebra 1 math · Lesson · Standards: F-IF.A.3, F-BF.A.2, F-LE.A.2

Quick answer: A sequence is a function on the counting numbers. An arithmetic sequence adds a common difference d each step, so $a_n = a_1 + (n - 1)d$. A geometric sequence multiplies by a common ratio r , so $a_n = a_1 r^{n-1}$. Recursive formulas give each term from the previous one; explicit formulas jump straight to any term.

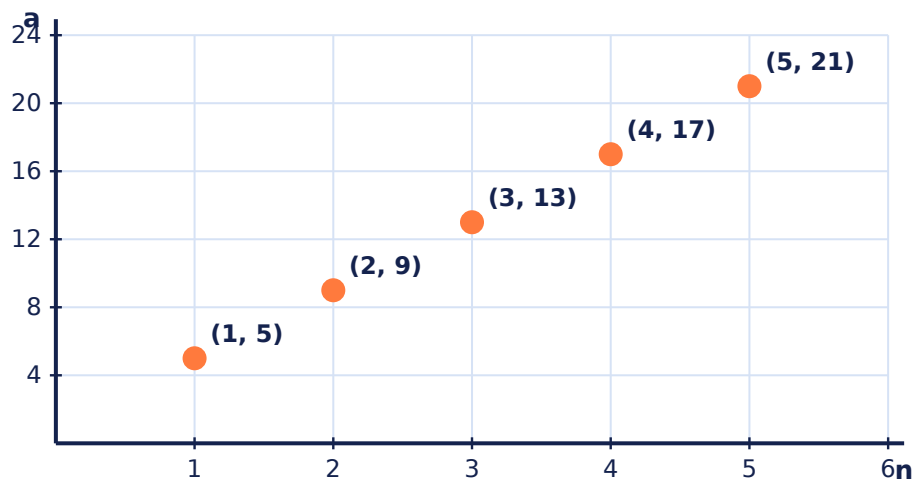
Standards: F-IF.A.3, F-BF.A.2, F-LE.A.2 · **Level:** Grade 9 Algebra 1

How is a sequence like a function?

Picture a numbered list. The position n goes in, and the **term** a_n comes out. Because only whole-number positions exist, a **sequence** graphs as separate dots rather than a connected line.

How does an arithmetic sequence work?

Every step of an **arithmetic sequence** is the same jump up or down. That jump is the **common difference** d .



Arithmetic: equal steps of +4 make the points lie on a line.

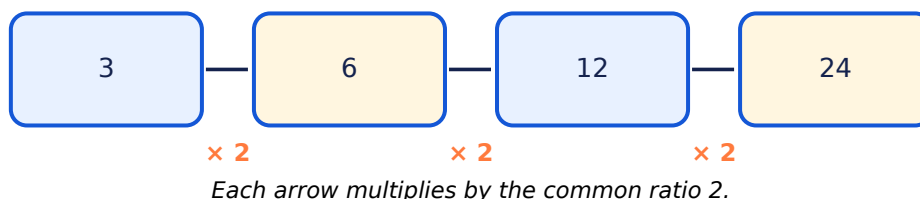
Arithmetic formulas

Recursive: a_1 given, $a_n = a_{n-1} + d$.

Explicit: $a_n = a_1 + (n - 1)d$.

How does a geometric sequence work?

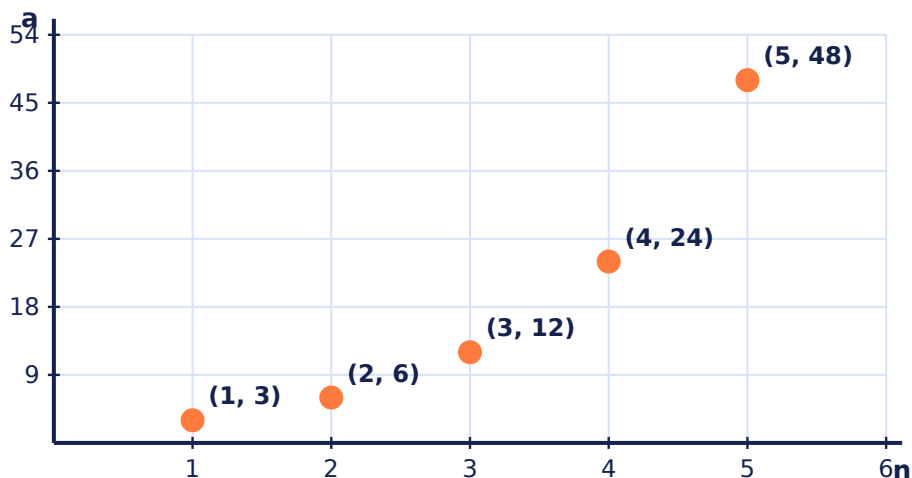
In a **geometric sequence** each term is the previous one scaled by a fixed factor, the **common ratio** r .



Geometric formulas

Recursive: a_1 given, $a_n = r \cdot a_{n-1}$.

Explicit: $a_n = a_1 \cdot r^{n-1}$.



Geometric: the factor 2 makes the points curve upward.

Feature	Arithmetic	Geometric
Rule	add d	multiply by r
Example	4, 11, 18, 25 ($d = 7$)	5, 15, 45, 135 ($r = 3$)
Graph	points on a line	points on a curve
Linked function type	linear	exponential

Method: find the 20th term of 9, 14, 19, 24, ...

1. Check the pattern: each term adds 5, so the sequence is arithmetic and $d = 5$.
2. Write the explicit formula: $a_n = 9 + (n - 1)(5) = 5n + 4$.
3. Substitute $n = 20$: $5(20) + 4 = \mathbf{104}$.

Common mistake

Using n instead of $n - 1$. The first term has no steps added, so a_n counts $n - 1$ steps.

Mathbot's tip

To classify a sequence, test differences first and ratios second. If neither is constant, the sequence is neither arithmetic nor geometric.

Key points to remember

- A sequence is a function whose domain is the positive integers.
- Arithmetic: constant difference d ; explicit formula $a_n = a_1 + (n - 1)d$.
- Geometric: constant ratio r ; explicit formula $a_n = a_1 r^{n-1}$.
- A recursive formula needs a starting value and a rule from the previous term.
- Arithmetic sequences grow like linear functions, geometric ones like exponentials.

Arithmetic and Geometric Sequences as Functions: frequently asked questions

How do you tell arithmetic from geometric?

Subtract consecutive terms. If the differences match, it is arithmetic. If not, divide consecutive terms; matching quotients mean geometric. If neither is constant, it is neither.

What is the difference between explicit and recursive formulas?

A recursive formula builds each term from the one before it and needs the first term. An explicit formula gives the n th term directly, so you can find the 50th term without listing the first 49.

Why is $n - 1$ in the explicit formulas?

The first term already exists, so reaching term n takes only $n - 1$ steps. For 9, 14, 19, ... the 20th term uses 19 steps of $+5$: $9 + 19(5) = 104$.

Why are sequence graphs separate points?

The domain contains only whole-number positions, since there is no term number 2.5. So the graph shows dots at $n = 1, 2, 3, \dots$ rather than a continuous line.

How do you find d or r from two terms that are not next to each other?

For arithmetic, divide the change in value by the number of steps. If $a_3 = 17$ and $a_7 = 37$, then $d = (37 - 17) \div 4 = 5$. For geometric, take a root of the quotient.