

Arithmetic and Geometric Sequences as Functions

Grade 9 – Algebra 1 math · Worksheet Answer Key · Standards: F-IF.A.3, F-BF.A.2, F-LE.A.2

Each answer key states the common difference or ratio first, then the formula and a check with a specific term. Compare the formula type with the type of growth: linear for arithmetic and exponential for geometric.

Standards: F-IF.A.3, F-BF.A.2, F-LE.A.2 · **Level:** Grade 9 Algebra 1

Problem 1 Common difference

$13 - 7 = 6$, $19 - 13 = 6$, $25 - 19 = 6$, so $d = 6$.

Next term: $25 + 6 = 31$.

$d = 6$; next term 31

Problem 2 Common ratio

$30 \div 6 = 5$, $150 \div 30 = 5$, $750 \div 150 = 5$, so $r = 5$.

Next term: $750 \times 5 = 3,750$.

$r = 5$; next term 3,750

Problem 3 List an arithmetic sequence

12, then $12 - 4 = 8$, $8 - 4 = 4$, $4 - 4 = 0$, $0 - 4 = -4$.

12, 8, 4, 0, -4

Problem 4 List a geometric sequence

2, $2 \times 5 = 10$, $10 \times 5 = 50$, $50 \times 5 = 250$.

2, 10, 50, 250

Problem 5 Classify sequences

(a) Differences 5, 7, 9 and ratios 2.25, 1.78, 1.56: neither.

(b) Ratios $12/24 = 1/2$, $6/12 = 1/2$, $3/6 = 1/2$: geometric.

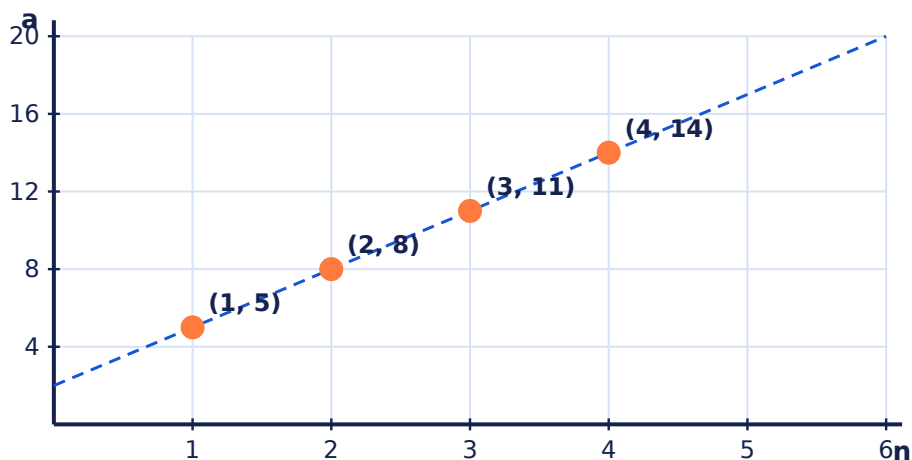
(c) Differences -3 , -3 , -3 : arithmetic.

(a) neither (b) geometric (c) arithmetic

Problem 6 Formula from a graph

The terms are 5, 8, 11, 14, so $d = 3$ and $a_1 = 5$.

$a_n = 5 + (n - 1)(3) = 3n + 2$. Then $a_{20} = 3(20) + 2 = 62$.



Each step up is $+3$, so $a_n = 3n + 2$.

$a_n = 3n + 2$; $a_{20} = 62$

Problem 7 Explicit arithmetic formula

$d = -4$ and $a_1 = 15$, so $a_n = 15 + (n - 1)(-4) = 19 - 4n$.

$a_{12} = 19 - 48 = -29$.

$a_n = 19 - 4n$; $a_{12} = -29$

Problem 8 Explicit geometric formula

$r = 3$ and $a_1 = 6$, so $a_n = 6 \cdot 3^{n-1}$.

$a_6 = 6 \cdot 3^5 = 6 \cdot 243 = 1,458$.

$a_n = 6 \cdot 3^{n-1}$; $a_6 = 1,458$

Problem 9 Recursive to explicit

9, 14, 19, 24, 29.

$$a_n = 9 + (n - 1)(5) = 5n + 4.$$

Check: $a_5 = 5(5) + 4 = 29$.

$$\mathbf{a_n = 5n + 4}$$

Problem 10 Recursive geometric

Each term is half the previous one: $a_1 = 80$, $a_n = 0.5 \cdot a_{n-1}$.

$$a_8 = 80 \cdot (1/2)^7 = 80 \div 128 = 0.625.$$

$$\mathbf{a_1 = 80, a_n = 0.5a_{n-1}; a_8 = 0.625}$$

Problem 11 Which term equals 128?

$$d = 6, \text{ so } a_n = 8 + 6(n - 1) = 6n + 2.$$

$$6n + 2 = 128 \text{ gives } 6n = 126 \text{ and } n = 21.$$

Check: $6(21) + 2 = 128$.

The 21st term

Problem 12 Two terms of an arithmetic sequence

$$d = (37 - 17) \div (7 - 3) = 20 \div 4 = 5.$$

$$a_1 = 17 - 2(5) = 7, \text{ so } a_n = 7 + 5(n - 1) = 5n + 2.$$

Check: $a_3 = 17$, $a_7 = 37$.

$$\mathbf{d = 5; a_1 = 7; a_n = 5n + 2}$$

Problem 13 Two terms of a geometric sequence

From term 2 to term 5 the ratio is applied 3 times: $r^3 = 324 \div 12 = 27$, so $r = 3$.

$$a_1 = 12 \div 3 = 4.$$

Check: 4, 12, 36, 108, 324.

$$r = 3; a_1 = 4$$

Problem 14 Sequence as a function

The terms are 2, 6, 18, 54, so $r = 6 \div 2 = 3$.

$$f(n) = 2 \cdot 3^{n-1}. \text{ Then } f(5) = 2 \cdot 3^4 = 2 \cdot 81 = 162.$$

The domain is the positive integers 1, 2, 3, ...

$$\mathbf{f(n) = 2 \cdot 3^{n-1}; f(5) = 162}$$

Problem 15 Weekly savings

$$a_1 = 15 \text{ and } d = 8, \text{ so } a_n = 15 + 8(n - 1) = 8n + 7.$$

$$8n + 7 > 150 \text{ gives } 8n > 143, \text{ so } n > 17.875.$$

Week 18 gives $8(18) + 7 = 151$; week 17 gives 143.

Week 18

Problem 16 Stadium rows

$$a_n = 22 + 3(n - 1) = 3n + 19.$$

$$\text{Row 30: } 22 + 3(29) = 109 \text{ seats.}$$

$$3n + 19 = 70 \text{ gives } 3n = 51 \text{ and } n = 17.$$

109 seats in row 30; row 17 has 70 seats

Problem 17 Viral video views

$$a_n = 3 \cdot 3^{n-1} = 3^n.$$

$$\text{Day 7: } 3^7 = 2,187.$$

$$3^8 = 6,561 \text{ is too small and } 3^9 = 19,683 \text{ exceeds } 10,000.$$

Day 1: 3

$$\text{Day 2: } 3 \times 3 = 9$$

$$\text{Day 3: } 9 \times 3 = 27$$

$$\text{Day } n: 3^n$$

Each day multiplies by 3.

$a_n = 3^n$; 2,187 views on day 7; day 9

Problem 18 Two depreciation models

Model A is geometric with $r = 0.8$: $a_4 = 18,000(0.8)^3 = 18,000(0.512) = 9,216$.

Model B is arithmetic: $18,000 - 3(2,500) = 10,500$.

Recursive formula for A: $a_1 = 18,000$, $a_n = 0.8 \cdot a_{n-1}$.

A: \$9,216; B: \$10,500; Model A is lower

Problem 19 Toothpick squares

1 square needs 4 toothpicks and each additional square adds 3, so $a_n = 4 + 3(n - 1) = 3n + 1$.

25 squares: $3(25) + 1 = 76$. For 100 toothpicks: $3n + 1 = 100$ gives $n = 33$.

1 square: 4 toothpicks
2 squares: $4 + 3 = 7$
3 squares: $4 + 3 + 3 = 10$
 n squares: $4 + 3(n - 1) = 3n + 1$

Each new square adds 3 toothpicks.

$a_n = 3n + 1$; 76 toothpicks; 33 squares

Problem 20 Bouncing ball

$h(n) = 160(0.75)^n$.

Bounce 5: $160(0.75)^5 = 160(0.2373046875) \approx 37.97$ inches.

Bounce 7: $160(0.75)^7 \approx 21.36$, still above 20. Bounce 8: $160(0.75)^8 \approx 16.02$, below 20.

$h(n) = 160(0.75)^n$; 37.97 inches; bounce 8