

Building Functions from Real Situations

Grade 9 – Algebra 1 math · Lesson · Standards: F-BF.A.1a, F-BF.A.1b, F-BF.B.4a, F-LE.A.1b

Quick answer: To build a function, name the input and the output and describe how the output is calculated, with an explicit or a recursive rule. Functions can be added, subtracted and multiplied, as in profit = revenue – cost. The inverse of a linear function comes from solving $y = f(x)$ for x .

Standards: F-BF.A.1a, F-BF.A.1b, F-BF.B.4a, F-LE.A.1b · **Level:** Grade 9 Algebra 1

How do you build a function from a situation?

To build a function, decide what the input and output are, then describe how the output is calculated. A rental shop charges a \$25 fee plus \$4 per hour. With h hours as the input, the output is $C(h) = 25 + 4h$.

Ways to describe the same function

- **Explicit:** $C(h) = 25 + 4h$, which gives any output directly.
- **Recursive:** $C(0) = 25$ and $C(h) = C(h - 1) + 4$, the starting value plus a rule for the next one.
- **Steps:** multiply hours by 4, then add 25.

When is a function linear?

A function is linear when its output changes by a constant amount for each equal step in the input, which is a constant rate per unit interval. The table $0 \rightarrow 5$, $2 \rightarrow 11$, $4 \rightarrow 17$ changes by 6 for every 2 units, so the rate is 3 per unit and $y = 3x + 5$.

How do you combine functions?

Arithmetic with functions

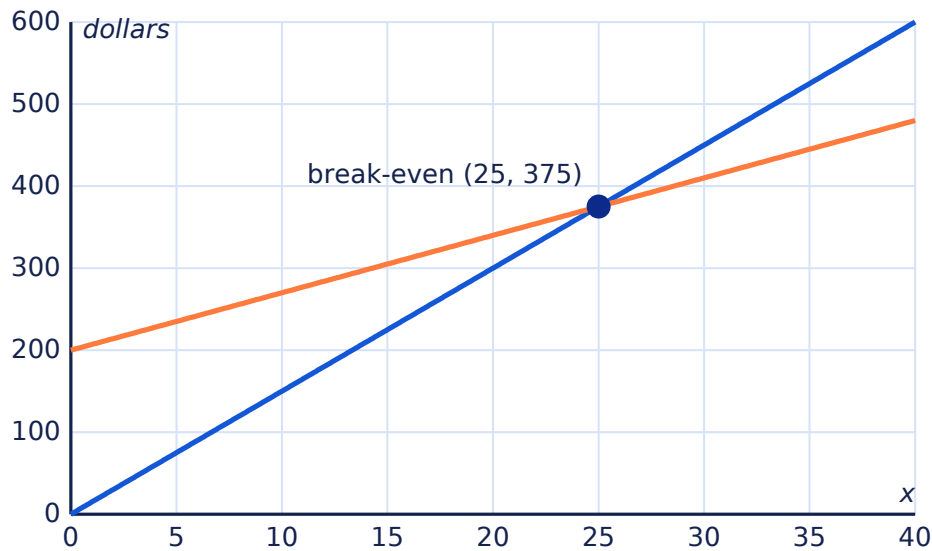
$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

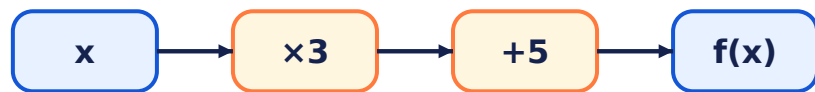
In business, profit is revenue minus cost: $P(x) = R(x) - C(x)$. With $R(x) = 15x$ and $C(x) = 200 + 7x$,

the profit is $8x - 200$. It is zero at the break-even point $x = 25$.

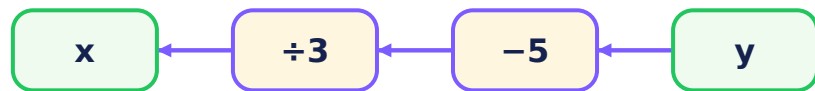


Revenue and cost meet at the break-even point.

How do you find the inverse of a linear function?



f: multiply by 3, then add 5



inverse: undo in reverse order (subtract 5, then divide by

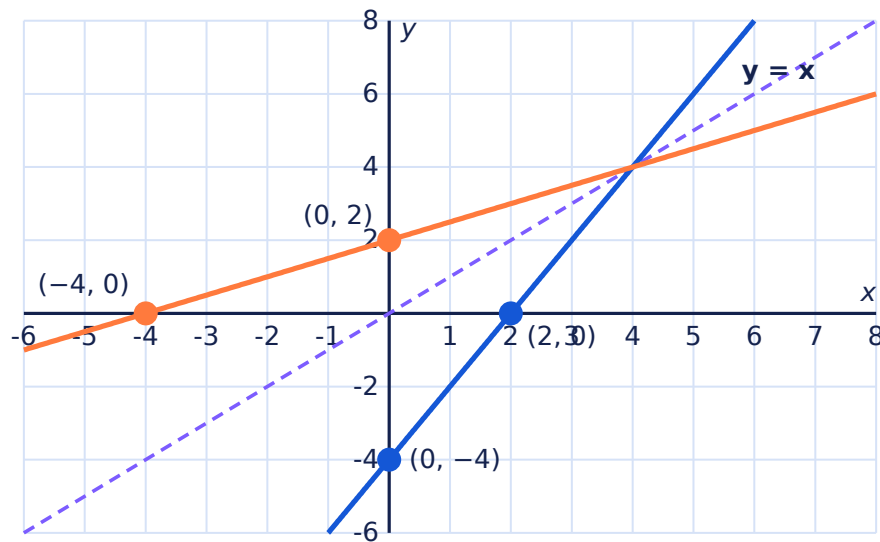
$f(x) = 3x + 5$ and its inverse, step by step.

Method

1. Write $y = f(x)$.
2. Swap the roles: solve the equation for x in terms of y .
3. Rename the variable: write the result as $f^{-1}(x)$.
4. Check that $f(f^{-1}(x)) = x$.

Worked example

$f(x) = 2x - 4$. Set $y = 2x - 4$, so $x = \frac{(y+4)}{2}$. The inverse is $f^{-1}(x) = \frac{(x+4)}{2}$. Check: $f(2) = 0$ and $f^{-1}(0) = 2$.



The graph of an inverse is the mirror image over $y = x$.

Common mistake

The symbol $f^{-1}(x)$ does not mean $1 \div f(x)$. It names the function that undoes f .

Mathbot's tip

To undo a recipe, reverse the steps and the operations: if f multiplies and then adds, the inverse subtracts and then divides.

Key points to remember

- Explicit rules give any output directly; recursive rules build each output from the previous one.
- A constant rate of change per unit interval gives a linear function.
- Profit = revenue – cost, so $P(x) = R(x) - C(x)$.
- The inverse of a linear function undoes it: solve $y = f(x)$ for x .
- The graph of an inverse is the mirror image of f over the line $y = x$.

Building Functions from Real Situations: frequently asked questions

What is the difference between an explicit and a recursive rule?

An explicit rule such as $f(n) = 25 + 4n$ computes any output directly from n . A recursive rule gives the starting value and tells how to get each term from the one before it.

How do you know a situation is linear?

The quantity changes by the same amount for each equal step in the input, such as \$4 per hour. That constant rate is the slope of the linear function.

How do you combine two functions?

Add, subtract or multiply their outputs for the same input. For example, profit is revenue minus cost, so $P(x) = R(x) - C(x)$.

How do you find the inverse of a linear function?

Write $y = f(x)$, solve for x , and then write the answer as a function of x . The inverse undoes the steps of f in the reverse order.

What does it mean to solve $f(x) = c$?

It means finding the input x that produces the output c . For a linear function this is the same as evaluating its inverse at c .