

Exponential Growth and Decay Models

Grade 9 – Algebra 1 math · Lesson · Standards: F-LE.A.1a, F-LE.A.1c, F-LE.A.2, F-LE.A.3, F-IF.C.8b, F-LE.B.5

Quick answer: An exponential function has the form $y = a \cdot b^x$, where a is the starting value and b is the factor applied each time x increases by 1. When b is greater than 1 the quantity grows, and when b is between 0 and 1 it decays. Linear functions add equal amounts, and exponential functions multiply by equal factors.

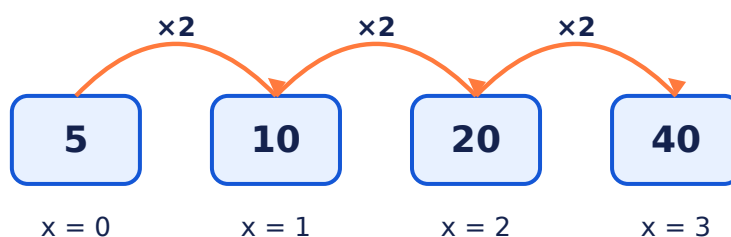
Standards: F-LE.A.1a, F-LE.A.1c, F-LE.A.2, F-LE.A.3, F-IF.C.8b, F-LE.B.5 · **Level:** Grade 9 Algebra 1

What makes a function exponential?

A linear function adds the same amount each time x goes up by 1. An exponential function multiplies by the same factor each time. Compare a savings jar that gains \$8 per week with a viral video whose views double every day.

Exponential function

A function of the form $y = a \cdot b^x$ with $a \neq 0$, $b > 0$ and $b \neq 1$. The number a is the initial value (the y -value at $x = 0$). The number b is the growth or decay factor.



Equal steps in x give the same multiplier: $\times 2$ every time.

How do you tell linear from exponential in a table?

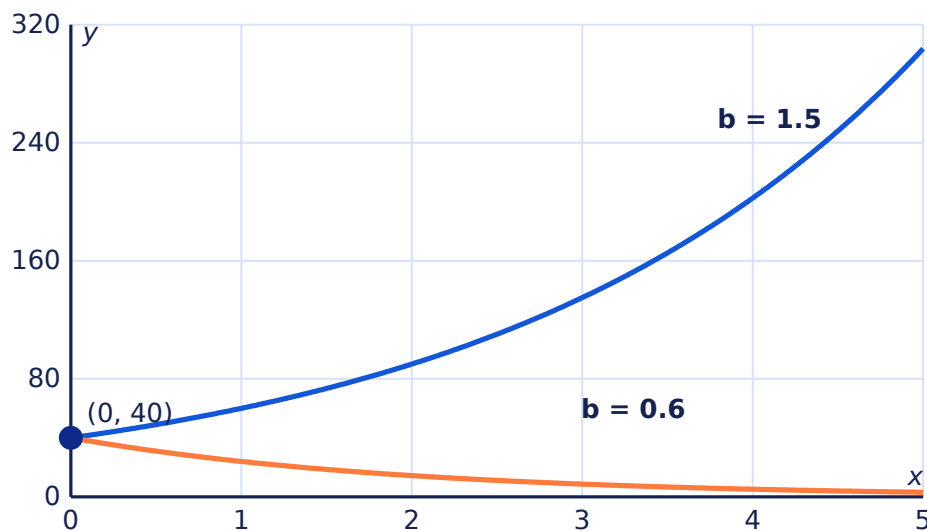
Look at equal steps in x . Equal differences in y mean linear. Equal ratios (each y divided by the one before it) mean exponential.

Type	Check	Equation
Linear	y -differences are equal	$y = mx + c$

Type	Check	Equation
Exponential	y-ratios are equal	$y = a \cdot b^x$

What do growth and decay look like?

If $b > 1$ the graph rises; if $0 < b < 1$ it falls toward the x-axis without touching it. Write $b = 1 + r$ for a growth rate r , and $b = 1 - r$ for a decay rate r .



Both curves start at 40. A factor above 1 grows; a factor between 0 and 1 decays.

Compound interest

With interest added once per year, $A = P(1 + r)^t$. Here P is the starting amount, r is the yearly rate as a decimal and t is the number of years.

How do you write an exponential model?

Method

1. Find the initial value a (the value when $x = 0$).
2. Divide any y-value by the previous one to get b .
3. Write $y = a \cdot b^x$ and test it on a second point.

Worked example

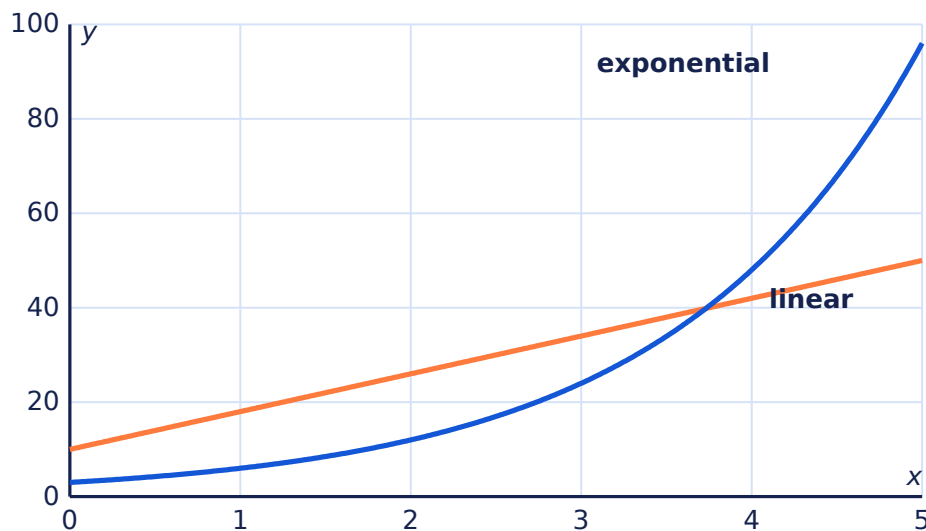
A trail camera uploads 6, 9, 13.5 photos on days 0, 1, 2. Then $a = 6$ and $b = 9 \div 6 = 1.5$, so $y = 6(1.5)^x$. Check: $6(1.5)^2 = 13.5$.

Common mistake

A 20% decay is $b = 0.8$, not 0.2. Subtract the rate from 1.

Why does an exponential eventually beat a line?

A line gains a fixed amount, while a curve with $b > 1$ gains more each step. However steep the line, some value of x exists where the curve is higher for good.



A line adds 8 each step; the curve multiplies by 2 each step and overtakes it.

Mathbot's tip

Ask "plus or times?" Same amount added means linear. Same multiplier means exponential.

Key points to remember

- Linear: equal differences in y . Exponential: equal ratios in y .
- In $y = a \cdot b^x$, a is the initial value and b is the factor per step.
- Growth rate r gives $b = 1 + r$; decay rate r gives $b = 1 - r$.
- Compound interest once a year: $A = P(1 + r)^t$.
- Any exponential with b above 1 eventually exceeds any linear function.

Exponential Growth and Decay Models: frequently asked questions

How can you tell if a table is exponential?

Divide each y -value by the one before it for equal steps in x . If every ratio is the same number, the table is exponential and that number is the factor b .

What does the number a mean in $y = a \cdot b^x$?

It is the initial value, the y -value when $x = 0$. On a graph it is the y -intercept, the point where the curve crosses the y -axis.

How do you turn a percent rate into the factor b ?

For growth, add the rate (as a decimal) to 1, so 7% growth gives $b = 1.07$. For decay, subtract it from 1, so 7% decay gives $b = 0.93$.

Does an exponential graph ever touch the x-axis?

No. When a is positive, the values of $a \cdot b^x$ stay positive for every x . A decay curve gets closer to the x-axis forever but never reaches it.

Why does exponential growth beat linear growth?

A linear function adds the same amount every step. An exponential function adds a bigger amount every step, so it eventually passes any line, no matter how steep.