

Exponential Growth and Decay Models

Grade 9 – Algebra 1 math · Worksheet Answer Key · Standards: F-LE.A.1a, F-LE.A.1c, F-LE.A.2, F-LE.A.3, F-IF.C.8b, F-LE.B.5

Every answer shows the ratio, the equation and the check. Compare the final answer in bold with your own, then re-read the steps for any problem you missed. Keep an eye on $1 + r$ versus $1 - r$.

Standards: F-LE.A.1a, F-LE.A.1c, F-LE.A.2, F-LE.A.3, F-IF.C.8b, F-LE.B.5 · **Level:** Grade 9 Algebra 1

Problem 1 Spot the Pattern, Table One

Ratios: $10 \div 5 = 2$, $20 \div 10 = 2$, $40 \div 20 = 2$. The differences (5, 10, 20) are not equal.

The table is **exponential with factor 2**.

Problem 2 Spot the Pattern, Table Two

Differences: $12 - 7 = 5$, $17 - 12 = 5$, $22 - 17 = 5$. The ratios are not equal ($12/7 \neq 17/12$).

The table is **linear with a constant difference of 5**.

Problem 3 Name the Parts of a Growth Model

Compare with $y = a \cdot b^x$: $a = 6$ and $b = 1.4$. Since $1.4 > 1$ the function grows. Rate: $1.4 - 1 = 0.40$.

Initial value 6, factor 1.4, growth of 40% per step.

Problem 4 Name the Parts of a Decay Model

$a = 250$ and $b = 0.8$. Since $0 < 0.8 < 1$ the function decays. Rate: $1 - 0.8 = 0.20$.

Initial value 250, factor 0.8, decay of 20% per step.

Problem 5 Evaluate an Exponential Function

$f(0) = 5(1) = 5$. $f(1) = 5(4) = 20$. $f(2) = 5(16) = 80$. $f(3) = 5(64) = 320$.

5, 20, 80, 320

Problem 6 Write the Rule from a Table

Initial value $a = 8$. Factor: $12 \div 8 = 1.5$, $18 \div 12 = 1.5$, $27 \div 18 = 1.5$.

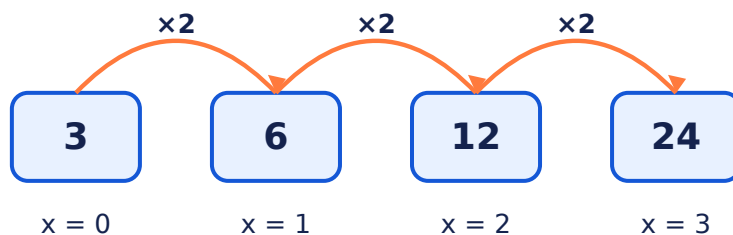
Check $x = 3$: $8(1.5)^3 = 8(3.375) = 27 \checkmark$.

$$y = 8(1.5)^x$$

Problem 7 Read an Exponential Graph

The y-intercept is 3, so $a = 3$. Factor: $6 \div 3 = 2$, $12 \div 6 = 2$, $24 \div 12 = 2$, so $b = 2$.

Equation: $y = 3(2)^x$. At $x = 4$: $3(16) = 48$.



Constant factor 2, starting value 3.

$$y = 3(2)^x; y = 48 \text{ when } x = 4$$

Problem 8 Table Without the Starting Value

Factor: $40 \div 80 = 0.5$, $20 \div 40 = 0.5$, $10 \div 20 = 0.5$, so $b = 0.5$ (decay).

Step back one: at $x = 0$ the value is $80 \div 0.5 = 160$.

Check $x = 3$: $160(0.5)^3 = 20 \checkmark$.

$$y = 160(0.5)^x$$

Problem 9 A Bank Rate Model

$b = 1.035 = 1 + 0.035$, so the rate is 3.5%.

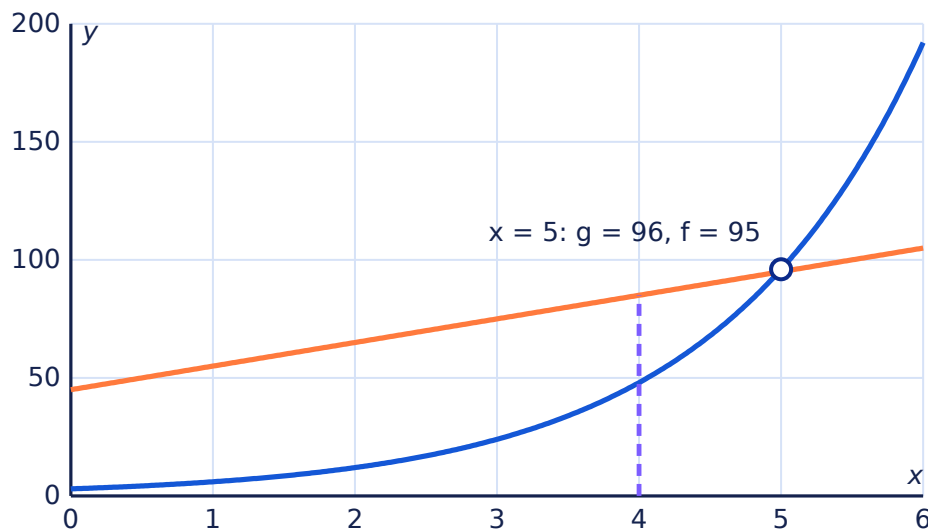
$$y = 1200(1.035)^2 = 1200(1.071225) = 1285.47.$$

3.5% per year; \$1,285.47 after 2 years

Problem 10 Line Versus Curve

Table: $x = 4$: $f = 85$, $g = 48$. $x = 5$: $f = 95$, $g = 96$.

For $x = 0$ to 4 , $g < f$ (3, 6, 12, 24, 48 against 45, 55, 65, 75, 85). At $x = 5$ the curve is just higher.



The curve g passes the line f between $x = 4$ and $x = 5$.

$x = 5$

Problem 11 Annual Compound Interest

$A = 2400(1.03)^6$. Since $1.03^3 = 1.092727$, $1.03^6 = 1.092727^2 = 1.1940523\dots$

$A = 2400(1.1940523) = 2865.73$.

\$2,865.73

Problem 12 Depreciating Truck

Decay factor: $1 - 0.12 = 0.88$, so $V = 18500(0.88)^t$.

$0.88^3 = 0.681472$, and $18500 \times 0.681472 = 12607.232$.

$V = 18,500(0.88)^t$; about \$12,607.23

Problem 13 Bacteria in a Lab Dish

$N = 300(2)^h$. Table: $h = 5$ gives $300 \times 32 = 9,600$; $h = 6$ gives $300 \times 64 = 19,200$.

$9,600 < 10,000 < 19,200$.

$$N = 300(2)^h; \text{ after 6 hours}$$

Problem 14 Interpret a Follower Model

1,800 is the starting number of followers ($t = 0$). 1.06 means the count grows by 6% each week.

$$F(10) = 1800(1.06)^{10} = 1800(1.790848...) = 3223.5..., \text{ which rounds to 3,224.}$$

1,800 starting followers; 6% weekly growth; about 3,224 followers

Problem 15 Half-Life of a Medicine

Halve repeatedly: 640 $\rightarrow$ 320 (6 h) $\rightarrow$ 160 (12 h) $\rightarrow$ 80 (18 h) $\rightarrow$ 40 (24 h) $\rightarrow$ 20 (30 h) $\rightarrow$ 10 (36 h) $\rightarrow$ 5 (42 h).

After 18 hours: 80 mg. At 36 hours the amount is exactly 10 mg, not less than 10 mg, so the first time below 10 mg is 42 hours.

80 mg; 42 hours

Problem 16 Compounding Four Times a Year

Quarterly rate: $6\% \div 4 = 1.5\%$. Number of quarters: $4 \times 2 = 8$.

$$A = 5000(1.015)^8 = 5000(1.126493...) = 5632.46.$$

\$5,632.46

Problem 17 Find the Growth Rate from Data

$8,400 \div 8,000 = 1.05$ and $8,820 \div 8,400 = 1.05$, so the rate is 5%.

$$P = 8000(1.05)^t. P(4) = 8000(1.21550625) = 9724.05.$$

5% per year; $P = 8,000(1.05)^t$; about 9,724 people

Problem 18 Two Savings Plans

Plan A: $A(n) = 100 + 25n$. Plan B: $B(n) = 100(1.2)^n$.

$n = 3$: $A = 175$, $B = 172.80$. $n = 4$: $A = 200$, $B = 207.36$.

At month 3 Plan A is ahead; at month 4 Plan B is ahead.

Month 4

Problem 19 Which Amount Is Larger?

$$A(n) = 25 + 15n; B(n) = 2^n.$$

Day 6: $A = 115$, $B = 64$. Day 7: $A = 130$, $B = 128$. Day 8: $A = 145$, $B = 256$.

Day 7 is still lower for B ($128 < 130$).

Day 8

Problem 20 Phone Value Over Time

$V = 900(0.85)^t$. Half the price is \$450.

$$0.85^4 = 0.52200625 \rightarrow \$469.81. \quad 0.85^5 = 0.4437... \rightarrow \$399.33.$$

At 4 years the value is above \$450; at 5 years it is below.

$V = 900(0.85)^t$; after 5 years