

Factoring Polynomials with Patterns and Grouping

Grade 9 – Algebra 1 math · Lesson · Standards: A-SSE.A.2, A-SSE.B.3a

Quick answer: Factoring rewrites a polynomial as a product of simpler polynomials. Start by taking out the greatest common factor, then look for a pattern: a difference of squares, a perfect square trinomial, a trinomial with two integers that fit, or grouping. Factoring helps solve equations because a product is zero only when one of its factors is zero.

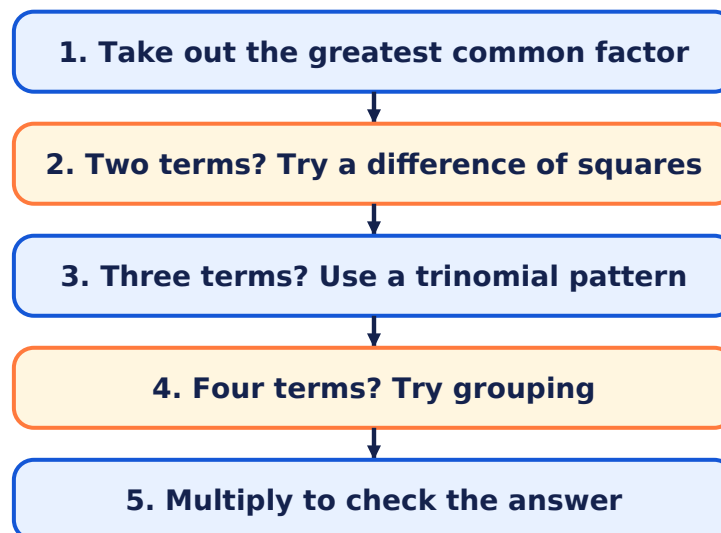
Standards: A-SSE.A.2, A-SSE.B.3a · **Level:** Grade 9 Algebra 1

What does it mean to factor a polynomial?

Factoring rewrites a sum as a product. Because $6x^2 + 9x = 3x(2x + 3)$, the product form shows the pieces that multiply together, and it shows where a graph crosses the x-axis.

Greatest common factor (GCF)

The largest monomial that divides every term. Take it out first, every time: $12x^3 + 18x^2 = 6x^2(2x + 3)$.



A factoring checklist: always start with the GCF.

How do you factor $x^2 + bx + c$?

Find two integers whose product is c and whose sum is b . For $x^2 + 7x + 12$ the pair 3 and 4 works,

so the factors are $(x + 3)(x + 4)$.

	x	4
x	x^2	$4x$
3	$3x$	12

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

How do you factor when the leading coefficient is not 1?

Method: split the middle term

1. Multiply a and c . For $6x^2 + 11x - 10$ that is -60 .
2. Find two numbers with product -60 and sum 11 : 15 and -4 .
3. Split: $6x^2 + 15x - 4x - 10$.
4. Group and factor each pair: $3x(2x + 5) - 2(2x + 5)$.
5. Write the shared factor once: **$(2x + 5)(3x - 2)$** .

Which patterns factor quickly?

Special patterns

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

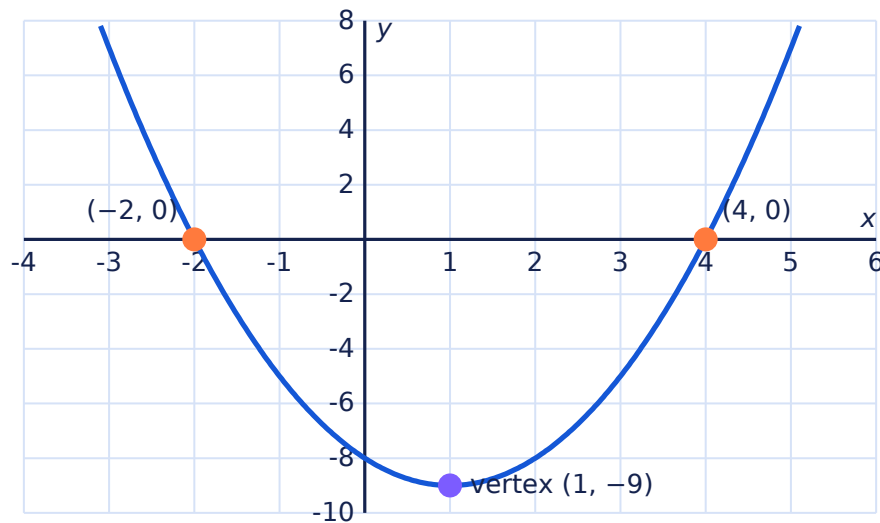
Four terms often factor by grouping: $x^3 + 4x^2 + 3x + 12 = x^2(x + 4) + 3(x + 4) = (x^2 + 3)(x + 4)$.

Common mistake

A sum of squares such as $x^2 + 16$ does not factor over the real numbers, so do not write $(x + 4)^2$.

How does factoring find zeros?

If a product equals zero, one of its factors is zero (the zero-product property). To solve $x^2 - 2x - 8 = 0$, factor to $(x - 4)(x + 2) = 0$, so $x = 4$ or $x = -2$. These are the x -intercepts of the graph.



Zeros of $y = (x - 4)(x + 2)$ are $x = -2$ and $x = 4$.

Mathbot's tip

Always multiply your factors back out. If you do not get the original expression, one sign is wrong.

Key points to remember

- Take out the greatest common factor before anything else.
- For $x^2 + bx + c$, find two integers with product c and sum b .
- $a^2 - b^2 = (a + b)(a - b)$ and $a^2 \pm 2ab + b^2 = (a \pm b)^2$.
- Four terms often factor by grouping in pairs.
- Zero-product property: if $(x - r)(x - s) = 0$ then $x = r$ or $x = s$.

Factoring Polynomials with Patterns and Grouping: frequently asked questions

What should you try first when factoring?

Look for a greatest common factor of all the terms and take it out. Doing this first makes the remaining polynomial smaller and easier to factor.

How do you factor $x^2 + bx + c$?

Find two integers that multiply to c and add to b . Then write $(x + p)(x + q)$ using those two integers. Multiply back to check the middle term.

Why can you not factor $x^2 + 25$?

A sum of two squares has no pair of real numbers that multiply to 25 and add to 0. Only a difference of squares, such as $x^2 - 25$, factors.

What is the zero-product property?

If two numbers multiply to give zero, at least one of them is zero. For $(x - 3)(x + 7) = 0$, set each factor equal to zero to get $x = 3$ or $x = -7$.

How can you check a factoring answer?

Multiply the factors using FOIL or an area model. The product must match the original polynomial exactly, including every sign.