

Shifting, Stretching and Piecewise Functions

Grade 9 – Algebra 1 math · Lesson · Standards: F-BF.B.3, F-IF.C.7b

Quick answer: Transformations change a parent graph: $f(x) + k$ shifts it vertically, $f(x - h)$ shifts it horizontally, and $a \cdot f(x)$ stretches or flips it. In $y = a|x - h| + k$ the vertex is (h, k) . Piecewise functions use a different rule on each interval, while step functions use flat pieces.

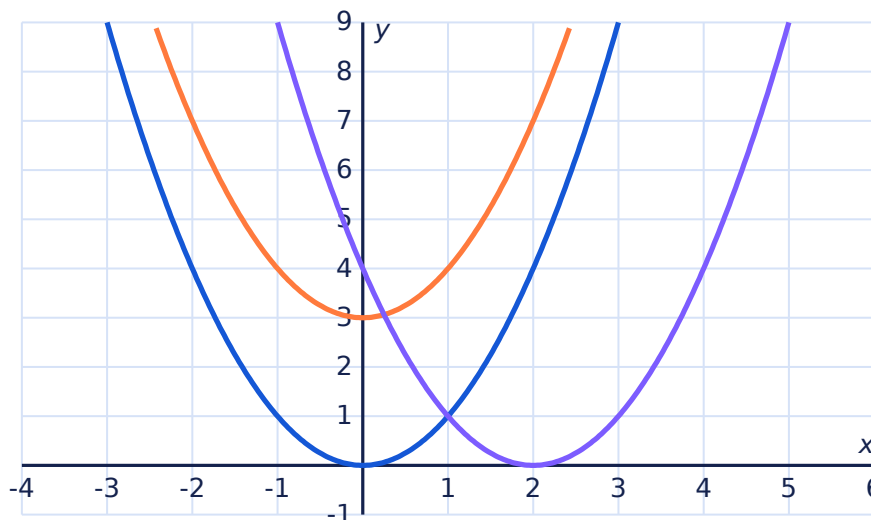
Standards: F-BF.B.3, F-IF.C.7b · **Level:** Grade 9 Algebra 1

How do you move a graph up, down, left or right?

Start with a parent function such as $f(x) = x^2$, $f(x) = |x|$ or $f(x) = x$. Changing the rule transforms the graph without changing its basic shape.

Transformations of $y = f(x)$

- $f(x) + k$: moves the graph up k units (down if k is negative).
- $f(x - h)$: moves the graph right h units (left if h is negative).
- $a \cdot f(x)$: stretches vertically by $|a|$; a negative a also flips the graph over the x -axis.



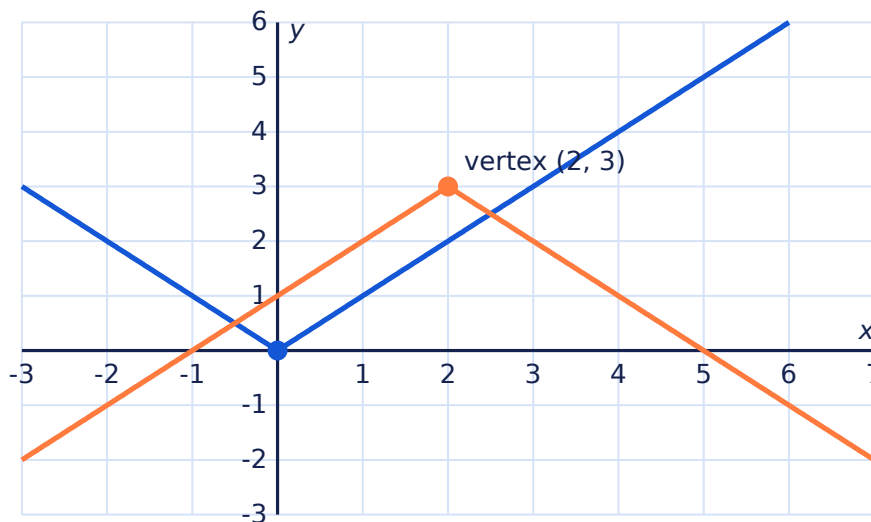
Adding 3 outside lifts the graph; subtracting 2 inside moves it right.

Common mistake

The change inside the parentheses works backward. $(x - 2)^2$ moves right 2, and $(x + 2)^2$ moves left 2.

What does an absolute value graph look like?

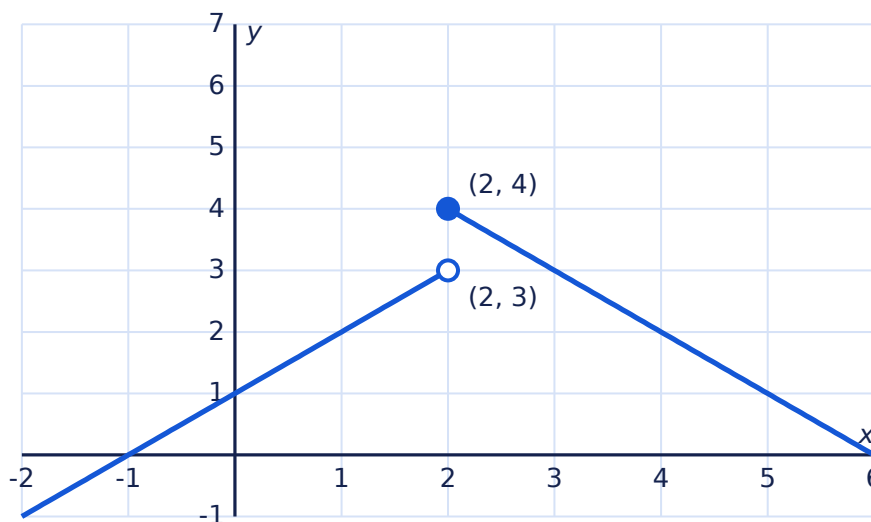
The absolute value $|x|$ is the distance from x to 0, so it is never negative. Its graph is a V with the vertex at the origin. In $y = a|x - h| + k$ the vertex is (h, k) , and the V opens down when $a < 0$.



$$y = |x| \text{ and } y = -|x - 2| + 3.$$

How do piecewise and step functions work?

A piecewise function uses different rules on different parts of the domain. To evaluate at a number, first decide which interval contains it, then use only that rule. Open circles mark endpoints that are not included, and closed circles mark endpoints that are included.



An open circle excludes a point; a closed circle includes it.

Step function

A piecewise function whose pieces are horizontal segments, so the graph looks like stairs.

Parking fees and shipping prices often work this way: any part of an hour or a pound is charged

as a whole one.

How do you graph a transformation step by step?

Method

1. Name the parent function and plot its key points.
2. Apply the stretch or flip (multiply the y -values by a).
3. Apply the horizontal shift, then the vertical shift.
4. Check one point by substituting its x -value in the new rule.

Worked example

$g(x) = -2|x + 1| + 3$ has vertex $(-1, 3)$ and opens down. At $x = 1$: $-2(2) + 3 = -1$, so $(1, -1)$ is on the graph.

Mathbot's tip

Track the vertex first. It tells you the shift, and the sign of a tells you whether the graph opens up or down.

Key points to remember

- $f(x) + k$ moves the graph up k units; $f(x - h)$ moves it right h units.
- $a \cdot f(x)$ stretches vertically by $|a|$ and flips over the x -axis if a is negative.
- In $y = a|x - h| + k$ the vertex is (h, k) .
- For a piecewise function, find the interval first, then use its rule.
- A step function has horizontal pieces, with open and closed endpoints.

Shifting, Stretching and Piecewise Functions: frequently asked questions

Why does $f(x - 3)$ move the graph right?

The output $f(x - 3)$ equals $f(0)$ when $x = 3$. Every point of the original graph now appears 3 units further to the right, so the shift is opposite to the sign in the parentheses.

What does a negative a do in $a \cdot f(x)$?

It multiplies every output by a negative number, which flips the graph over the x -axis. A parabola that opened up now opens down, and its vertex becomes a maximum.

How do you find the vertex of $y = a|x - h| + k$?

The vertex is the point (h, k) . The expression inside the absolute value is zero when $x = h$, so that is where the V turns, at height k .

How do you evaluate a piecewise function?

Check which interval the input belongs to, then substitute into the rule for that interval only. At a boundary, use the rule whose inequality includes the endpoint.

What do open and closed circles mean on a graph?

A closed circle means the point is part of the graph. An open circle means the graph approaches that point, but the point itself is not included.