

Shifting, Stretching and Piecewise Functions

Grade 9 – Algebra 1 math · Worksheet Answer Key · Standards: F-BF.B.3, F-IF.C.7b

Each answer names the transformation or rule that applies and shows the arithmetic, with the final result in bold. When you compare with a graph, check the vertex first. For piecewise problems, confirm which interval holds the input before using a rule.

Standards: F-BF.B.3, F-IF.C.7b · **Level:** Grade 9 Algebra 1

Problem 1 Shift Up

Adding 5 outside the function raises every point 5 units. The vertex (0, 0) moves to (0, 5).

Up 5 units; vertex (0, 5)

Problem 2 Shift Right

Subtracting 4 inside the parentheses moves the graph 4 units to the right. The vertex is (4, 0).

Right 4 units; vertex (4, 0)

Problem 3 Absolute Value, Shifted Down

The -3 is outside the absolute value, so the V moves down 3. The vertex moves from (0, 0) to (0, -3).

Down 3 units; vertex (0, -3)

Problem 4 Absolute Value, Shifted Left

$x + 2 = x - (-2)$, so the V moves 2 units left. The vertex is (-2 , 0).

Left 2 units; vertex (-2 , 0)

Problem 5 Evaluate an Absolute Value Function

$f(0) = |-7| = 7$. $f(3) = |-1| = 1$. $f(5) = |3| = 3$. $f(-1) = |-9| = 9$.

7, 1, 3, 9

Problem 6 Evaluate a Piecewise Function

$-3 < 0$, so use $x^2 - 1$: $9 - 1 = 8$. $0 \geq 0$, so use $3x + 2$: $2 \cdot 4 \geq 0$: $3(4) + 2 = 14$.

$$f(-3) = 8, f(0) = 2, f(4) = 14$$

Problem 7 Stretch, Shrink and Flip

- Every y -value is tripled: a vertical stretch by 3 (narrower).
- Every y -value is divided by 4: a vertical shrink (wider).
- Every y -value changes sign: a reflection over the x -axis.

(a) stretch by 3; (b) shrink by $\frac{1}{4}$; (c) reflection over the x -axis

Problem 8 Combine Several Changes

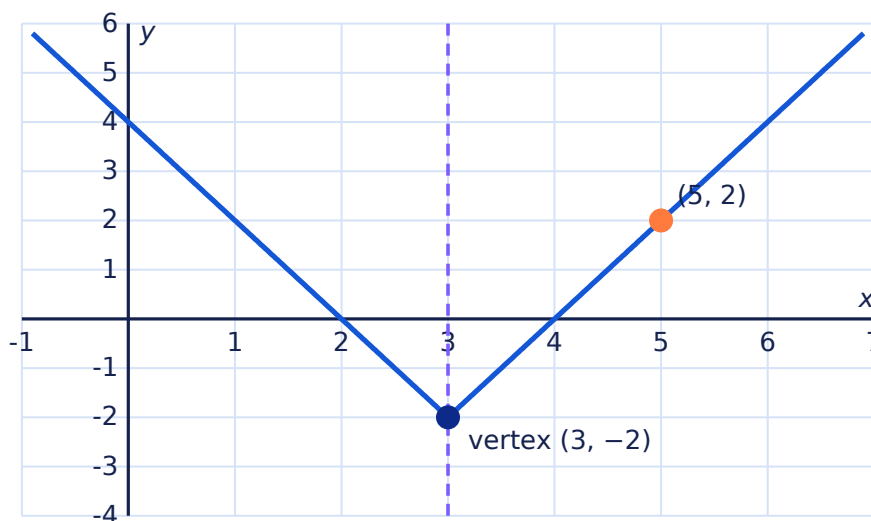
Right 3, stretch by 2, flip over the x -axis, up 5. The vertex is $(3, 5)$. Check: $g(3) = 5$ and $g(1) = -2(4) + 5 = -3$.

Because $a = -2$ is negative, the parabola opens down.

Vertex $(3, 5)$; opens down

Problem 9 Equation from an Absolute Value Graph

The vertex is $(3, -2)$, so $h = 3$ and $k = -2$. Use $(5, 2)$: $2 = a|5 - 3| - 2$, so $2a = 4$ and $a = 2$.



The point $(5, 2)$ fixes the stretch factor $a = 2$.

Check $(6, 4)$: $2|6 - 3| - 2 = 4 \checkmark$.

$$y = 2|x - 3| - 2$$

Problem 10 Table for $f(x - 2) + 1$

$$g(0) = f(-2) + 1 = 4 + 1 = 5. \quad g(2) = f(0) + 1 = 1. \quad g(4) = f(2) + 1 = 5.$$

The vertex (2, 1) is the lowest point.

(0, 5), (2, 1), (4, 5)

Problem 11 Read a Piecewise Graph

The left piece is $y = x + 1$: $f(-1) = 0$ and $f(1) = 2$. At $x = 2$ the closed circle is at (2, 4), so $f(2) = 4$. The right piece is $y = -x + 6$: $f(3) = 3$.

The open circle at (2, 3) belongs to the left piece and is not included.

0, 2, 4, 3; open at (2, 3), closed at (2, 4)

Problem 12 Absolute Value Equation

$$|x - 4| + 1 = 7, \text{ so } |x - 4| = 6. \text{ Then } x - 4 = 6 \text{ or } x - 4 = -6.$$

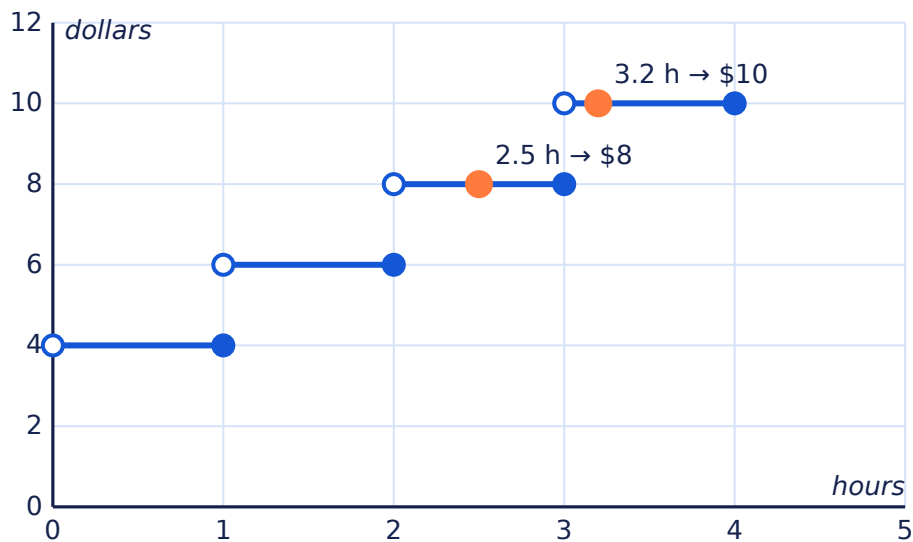
$x = 10$ or $x = -2$. Both are 6 units from 4 on the number line.

$x = 10$ or $x = -2$

Problem 13 Parking Fee Steps

1 hour is the right end of the first step, a closed circle at (1, 4): \$4.

2.5 hours lies in the step $2 < h \leq 3$: \$8. 3.2 hours lies in the step $3 < h \leq 4$: \$10.



Costs read from the steps.

\$4, \$8, \$10

Problem 14 Taxi Fare Rule

$C(m) = 3.50 + 2.25m$ for $0 \leq m \leq 4$, and $C(m) = 12.50 + 1.75(m - 4)$ for $m > 4$. (At 4 miles the fare is $3.50 + 9 = 12.50$.)

$$C(2) = 3.50 + 4.50 = 8. \quad C(10) = 12.50 + 1.75(6) = 23.$$

\$8.00 for 2 miles; \$23.00 for 10 miles

Problem 15 Move a Point

- Same x , $y + 4$: $(3, 9)$.
- Shift right 2: $(5, 5)$.
- Flip the y -value: $(3, -5)$.
- Triple the y -value: $(3, 15)$.

(a) (3, 9); (b) (5, 5); (c) (3, -5); (d) (3, 15)

Problem 16 Write the Transformed Parabola

Left 4: $(x + 4)^2$. Reflect: $-(x + 4)^2$. Up 7: $y = -(x + 4)^2 + 7$.

$$\text{At } x = -2: -(2)^2 + 7 = 3.$$

$$\mathbf{y = -(x + 4)^2 + 7; y = 3}$$

Problem 17 Absolute Value as Piecewise

The inside $x - 3$ is negative when $x < 3$ and non-negative when $x \geq 3$. So $f(x) = -(x - 3) = 3 - x$ for $x < 3$, and $f(x) = x - 3$ for $x \geq 3$.

$$x = -1: 3 - (-1) = 4 = |-4| \checkmark. x = 6: 6 - 3 = 3 = |3| \checkmark.$$

$$f(x) = 3 - x \text{ for } x < 3; f(x) = x - 3 \text{ for } x \geq 3$$

Problem 18 Electricity Bill

$C(k) = 0.12k$ for $0 \leq k \leq 500$, and $C(k) = 60 + 0.18(k - 500)$ for $k > 500$. At 500 kWh both rules give \$60, so the graph has no gap.

$$C(430) = 0.12(430) = 51.60. C(760) = 60 + 0.18(260) = 60 + 46.80 = 106.80.$$

\$51.60 and \$106.80

Problem 19 Describe a Transformed Absolute Value

Left 1, stretch by 3, flip over the x -axis, up 2. The vertex is $(-1, 2)$ and the V opens down.

$g(-4) = -3|-3| + 2 = -9 + 2 = -7$. $g(2) = -3|3| + 2 = -7$. The values match because -4 and 2 are both 3 units from -1 .

Vertex $(-1, 2)$; $g(-4) = -7$ and $g(2) = -7$

Problem 20 Robot on a Track

The vertex is $(4, 0)$, so $h = 4$ and $k = 0$. At $t = 0$: $6 = a|0 - 4| = 4a$, so $a = 1.5$.

Check $t = 8$: $1.5(4) = 6 \checkmark$. The vertex means that the robot reaches the dock after 4 seconds, and the speed is 1.5 feet per second.

$$d = 1.5|t - 4|$$