

Solution Sets, Line-Parabola Systems and Exponentials

Grade 9 – Algebra 1 math · Lesson · Standards: A-REI.D.10, A-REI.C.7, F-IF.C.7e, A-SSE.B.3c

Quick answer: The graph of an equation is the set of all points that satisfy it. To solve a line and a parabola together, set their y-expressions equal and solve the quadratic, which gives zero, one or two intersection points. Exponential graphs show a y-intercept and a horizontal asymptote, and exponent rules turn a yearly rate into a monthly rate.

Standards: A-REI.D.10, A-REI.C.7, F-IF.C.7e, A-SSE.B.3c · **Level:** Grade 9 Algebra 1

What does the graph of an equation really show?

The graph of an equation in two variables is the set of all points (x, y) whose coordinates make the equation true. A point on the graph is a solution, and a point off the graph is not. For $y = x^2 - 5$, the point $(3, 4)$ is on the graph because $3^2 - 5 = 4$, but $(3, 5)$ is not.

How do you solve a line and a parabola together?

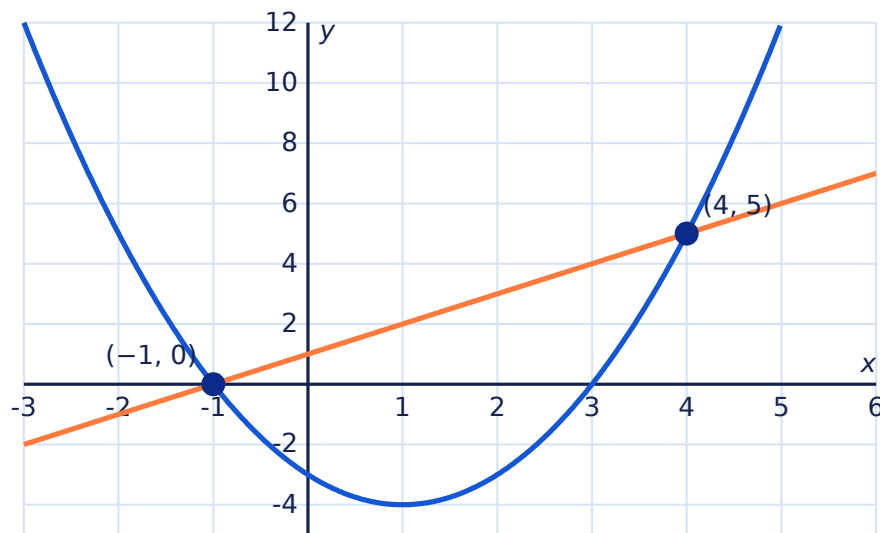
A solution of a system must make both equations true, so it is a point where the graphs cross. To find it exactly, set the two expressions for y equal and solve.

Method: substitution

1. Write both equations as $y = \dots$
2. Set the right sides equal to get a quadratic in x .
3. Solve for x (factor or use the quadratic formula).
4. Put each x into the linear equation to find y .

Worked example

$y = x^2 - 2x - 3$ and $y = x + 1$. Then $x^2 - 2x - 3 = x + 1$, so $x^2 - 3x - 4 = 0$ and $(x - 4)(x + 1) = 0$. So $x = 4$ or $x = -1$, giving the points $(4, 5)$ and $(-1, 0)$.

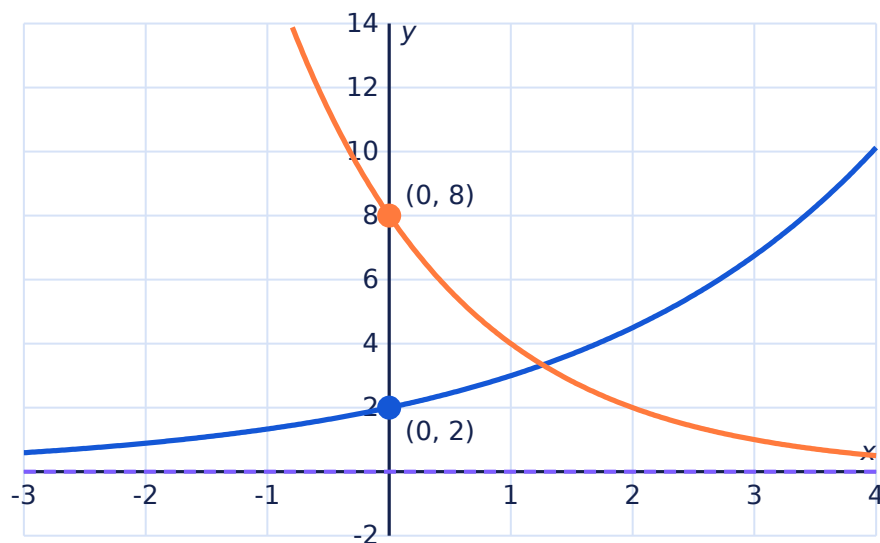


The graphs cross where both equations are true.

A line and a parabola can meet in two points, one point (tangent) or none. If the quadratic has a negative discriminant, there is no real solution.

How do you graph an exponential function?

For $y = a \cdot b^x$, the y-intercept is $(0, a)$. When $b > 1$ the graph rises to the right and approaches the x-axis on the left. When $0 < b < 1$ it falls toward the x-axis on the right. The line $y = 0$ is a horizontal asymptote, so there is no x-intercept.



Both curves approach the dashed line $y = 0$ but never touch it.

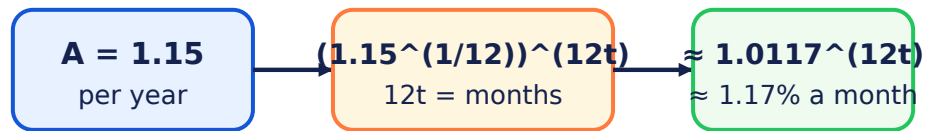
End behavior

$b > 1$ ($a > 0$): as $x \rightarrow \infty$, $y \rightarrow \infty$; as $x \rightarrow -\infty$, $y \rightarrow 0$.

$0 < b < 1$ ($a > 0$): as $x \rightarrow \infty$, $y \rightarrow 0$; as $x \rightarrow -\infty$, $y \rightarrow \infty$.

How do you rewrite an exponential expression with a different time unit?

Use the power rule $(b^m)^n = b^{mn}$. A balance that grows 15% a year follows 1.15^t with t in years. Since $t = (12t) \div 12$, $1.15^t = (1.15^{1/12})^{12t}$, and $1.15^{1/12} \approx 1.0117$. So the equivalent monthly growth rate is about 1.17%.



Rewriting 1.15^t as $(1.15^{(1/12)})^{(12t)}$.

Common mistake

Dividing 15% by 12 to get a monthly rate of 1.25%. Monthly growth compounds, so the monthly factor is the twelfth root of 1.15.

Mathbot's tip

Always check a solution in both equations. A point that fits only one is not a solution of the system.

Key points to remember

- A point is on the graph of an equation exactly when its coordinates satisfy the equation.
- To solve a linear-quadratic system, set the two y-expressions equal and solve for x.
- A line meets a parabola in 0, 1 or 2 points.
- The graph of $y = a \cdot b^x$ has y-intercept (0, a) and the horizontal asymptote $y = 0$.
- $(b^m)^n = b^{mn}$ lets you write 1.15^t as $(1.15^{(1/12)})^{(12t)}$ to find the monthly rate.

Solution Sets, Line-Parabola Systems and Exponentials: frequently asked questions

What does it mean for a point to be on a graph?

The point lies on the graph exactly when its coordinates make the equation true. Substitute x and y: if both sides are equal, the point is a solution of the equation.

How do you solve a system of a line and a parabola?

Set the two expressions for y equal, move everything to one side and solve the quadratic for x. Then substitute each x into the linear equation to find the matching y.

How many solutions can a line-parabola system have?

It can have two solutions when the line crosses the parabola twice, one when the line just touches it, and none when they never meet.

What is a horizontal asymptote?

It is a horizontal line that a graph gets closer and closer to without reaching. The graph of $y = a \cdot b^x$ has the asymptote $y = 0$, the x-axis.

How do you turn a yearly growth factor into a monthly one?

Raise the yearly factor to the power $1/12$. For 1.15 per year, $1.15^{(1/12)} \approx 1.0117$, which is about 1.17% growth per month. Dividing 15% by 12 is wrong.