

Solution Sets, Line-Parabola Systems and Exponentials

Grade 9 – Algebra 1 math · Worksheet Answer Key · Standards: A-REI.D.10, A-REI.C.7, F-IF.C.7e, A-SSE.B.3c

Each answer shows the substitution or equation that was solved, then the final result in bold. For systems, both coordinates of every solution are given and checked. Rates from exponent rules are rounded as requested in each problem.

Standards: A-REI.D.10, A-REI.C.7, F-IF.C.7e, A-SSE.B.3c · **Level:** Grade 9 Algebra 1

Problem 1 Test Two Points on a Line

$(3, 11)$: $2(3) + 5 = 11$, so it is on the graph. $(-1, 4)$: $2(-1) + 5 = 3$, not 4, so it is not.

$(3, 11)$ is on the graph; $(-1, 4)$ is not

Problem 2 Test Three Points on a Parabola

$(4, 10)$: $16 - 6 = 10$ ✓. $(-3, 3)$: $9 - 6 = 3$ ✓. $(2, -1)$: $4 - 6 = -2$, not -1 .

$(4, 10)$ and $(-3, 3)$

Problem 3 Complete the Solution Pair

a. $3(4) - 7 = 5$.

b. $(-3)^2 + 2(-3) = 9 - 6 = 3$.

(a) $(4, 5)$; (b) $(-3, 3)$

Problem 4 Growth y-Intercept and a Value

At $x = 0$: $5(1) = 5$, so the intercept is $(0, 5)$. At $x = 3$: $5(8) = 40$. Since $2 > 1$ the graph rises.

$(0, 5)$; 40; rises

Problem 5 Decay y-Intercept and a Value

At $x = 0$: 80. At $x = 3$: $80 \cdot \frac{1}{8} = 10$. The values halve each step and never reach 0, so the asymptote is $y = 0$.

(0, 80); 10; $y = 0$

Problem 6 Negative Inputs

$3^{-1} = \frac{1}{3}$, so $y = 6 \cdot \frac{1}{3} = 2$. $3^{-2} = \frac{1}{9}$, so $y = 6 \cdot \frac{1}{9} = \frac{2}{3}$.

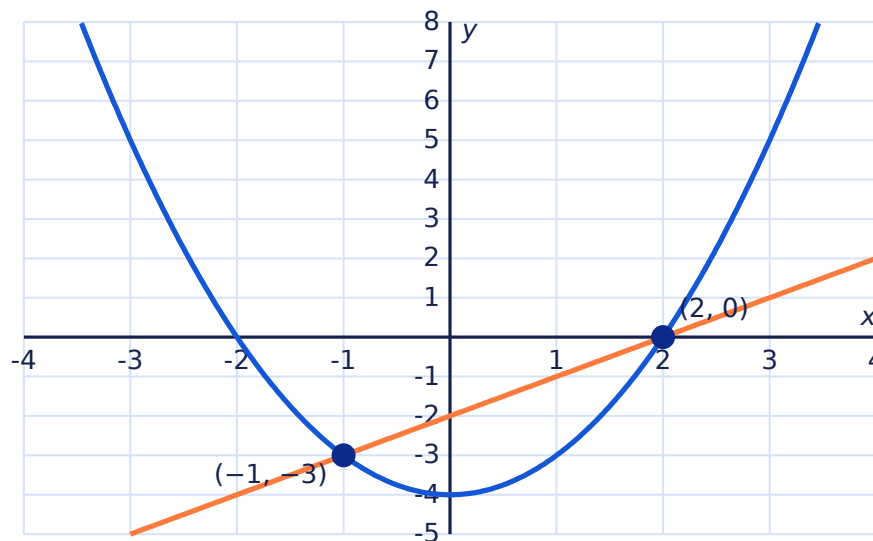
The values shrink toward 0 as x decreases.

2 and $\frac{2}{3}$

Problem 7 Find Intersections from a Graph

Set equal: $x^2 - 4 = x - 2$, so $x^2 - x - 2 = 0$ and $(x - 2)(x + 1) = 0$. Then $x = 2$ or $x = -1$.

$y = x - 2$ gives 0 and -3 .



The intersections are (2, 0) and (-1, -3).

(2, 0) and (-1, -3)

Problem 8 Substitution with $y = x^2$

$x^2 = 2x + 8$, so $x^2 - 2x - 8 = 0$ and $(x - 4)(x + 2) = 0$.

$x = 4$ gives $y = 16$. $x = -2$ gives $y = 4$.

(4, 16) and (-2, 4)

Problem 9 System with a Missing Constant Term

$x^2 - x - 6 = 2x - 6$, so $x^2 - 3x = 0$ and $x(x - 3) = 0$.

$x = 0$ gives $y = -6$. $x = 3$ gives $y = 0$.

(0, -6) and (3, 0)

Problem 10 Exactly One Solution

$x^2 - 6x + 10 = 2x - 6$, so $x^2 - 8x + 16 = 0$ and $(x - 4)^2 = 0$. Then $x = 4$ and $y = 2$.

The line touches the parabola at one point only, so it is tangent to it.

(4, 2); the line is tangent to the parabola

Problem 11 No Solution

$x^2 + 3 = 2x$, so $x^2 - 2x + 3 = 0$. The discriminant is $(-2)^2 - 4(1)(3) = 4 - 12 = -8$, which is negative.

Therefore the graphs never meet.

No real solution (discriminant -8)

Problem 12 Tangent Condition

Set $x^2 = 2x + c$, so $x^2 - 2x - c = 0$. One solution needs discriminant 0: $4 + 4c = 0$, so $c = -1$.

Then $x^2 - 2x + 1 = 0$, so $x = 1$ and $y = 1$.

$c = -1$; the point (1, 1)

Problem 13 Read an Exponential Graph

The curve crosses the y-axis at (0, 4). The values halve as x increases: 4, 2, 1, so $b = \frac{1}{2}$.

As $x \rightarrow \infty$ the values approach 0; as $x \rightarrow -\infty$ they grow without bound. The asymptote is $y = 0$.

(0, 4); $y = 0$; $\rightarrow 0$ and $\rightarrow \infty$; $y = 4(\frac{1}{2})^x$

Problem 14 Rewrite with the Power Rule

$1.08^t = (1.08^{1/12})^{12t}$. The twelfth root of 1.08 is about 1.0064.

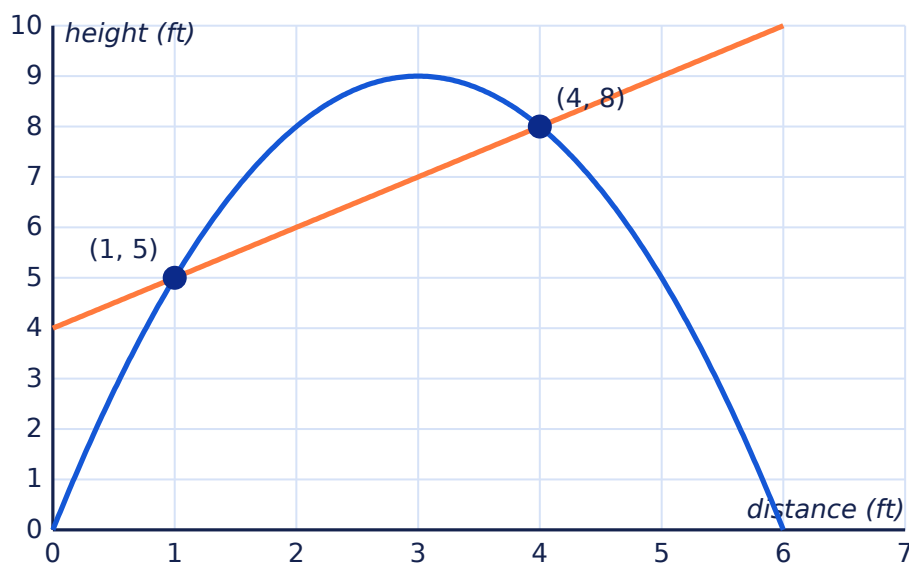
So $100(1.08)^t \approx 100(1.0064)^{12t}$, a growth of about 0.64% per month.

Monthly factor ≈ 1.0064

Problem 15 Ball Over a Ramp

$-x^2 + 6x = x + 4$, so $x^2 - 5x + 4 = 0$ and $(x - 1)(x - 4) = 0$. Then $x = 1$ or $x = 4$.

Heights: $h = 1 + 4 = 5$ and $h = 4 + 4 = 8$.



Intersections at (1, 5) and (4, 8).

(1 ft, 5 ft) and (4 ft, 8 ft)

Problem 16 Solve and Check Both Equations

$x^2 - 2x - 8 = -x + 4$, so $x^2 - x - 12 = 0$ and $(x - 4)(x + 3) = 0$.

$x = 4$: line gives $y = 0$; parabola $16 - 8 - 8 = 0$ ✓. $x = -3$: line gives $y = 7$; parabola $9 + 6 - 8 = 7$ ✓.

(4, 0) and (-3, 7)

Problem 17 Monthly Rate for a Savings Account

$A = 1500(1.06^{1/12})^{12t}$, and $1.06^{1/12} \approx 1.00487$, so the monthly rate is about 0.49%.

After 18 months, $t = 1.5$: $1500(1.06)^{1.5} = 1500(1.09134) \approx 1637.01$.

About 0.49% per month; \$1,637.01

Problem 18 Half-Life as Yearly Decay

$$\left(\frac{1}{2}\right)^{\frac{1}{5}} = \left(\left(\frac{1}{2}\right)^{\frac{1}{5}}\right)^t, \text{ so } b = \left(\frac{1}{2}\right)^{\frac{1}{5}} \approx 0.8706.$$

The yearly decay rate is $1 - 0.8706 = 0.1294$, about 12.9%.

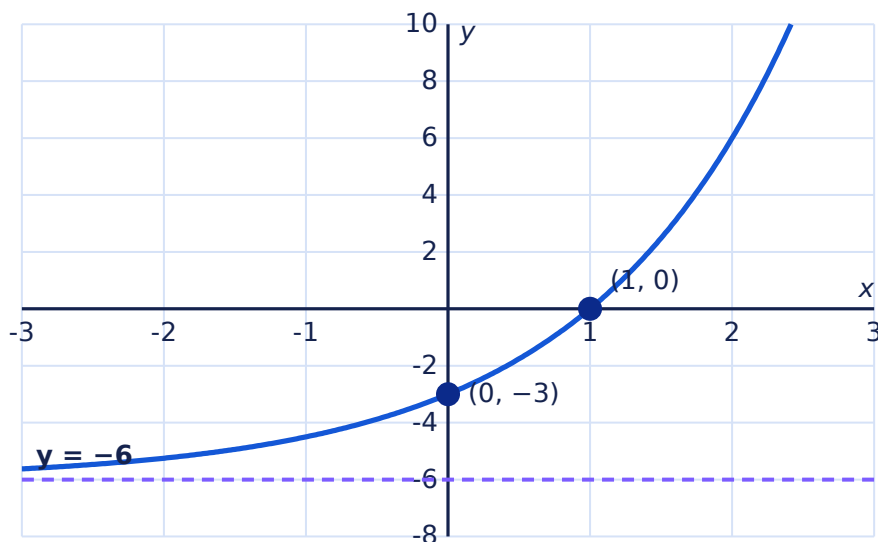
$$\text{At } t = 10: 200\left(\frac{1}{2}\right)^2 = 200 \cdot \frac{1}{4} = 50.$$

$b \approx 0.8706$; about 12.9% per year; 50 grams

Problem 19 Intercepts and End Behavior

$f(0) = 3 - 6 = -3$, so the y-intercept is $(0, -3)$. Set $3(2)^x - 6 = 0$: $2^x = 2$, so $x = 1$, and the x-intercept is $(1, 0)$.

The graph approaches $y = -6$ on the left. As $x \rightarrow \infty$, $f(x) \rightarrow \infty$; as $x \rightarrow -\infty$, $f(x) \rightarrow -6$.



$$f(x) = 3(2)^x - 6.$$

$(0, -3)$; $(1, 0)$; asymptote $y = -6$; $\rightarrow \infty$ and $\rightarrow -6$

Problem 20 Irrational Intersection Points

$x^2 = 2x + 1$, so $x^2 - 2x - 1 = 0$. The discriminant is $4 + 4 = 8$, so $x = \frac{(2 \pm \sqrt{8})}{2} = 1 \pm \sqrt{2}$.

$x \approx 2.41$ gives $y = 2(2.41) + 1 \approx 5.83$. $x \approx -0.41$ gives $y \approx 0.17$.

$x = 1 \pm \sqrt{2}$; points $\approx (2.41, 5.83)$ and $(-0.41, 0.17)$