

Key Features of Graphs and Rates of Change

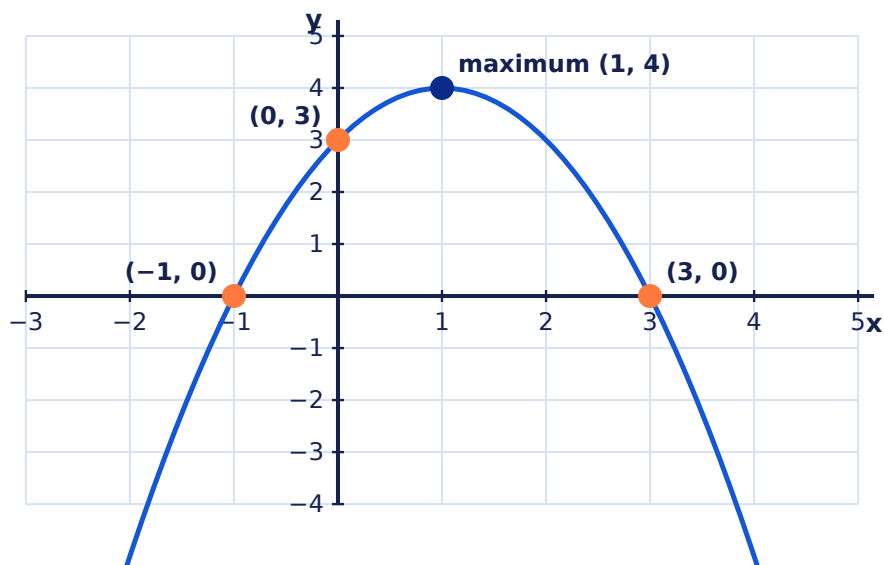
Grade 9 – Algebra 1 math · Lesson · Standards: F-IF.B.4, F-IF.B.6, F-IF.C.9

Quick answer: Graph features worth reading are intercepts, rising and falling intervals, relative peaks and valleys, sign, end behavior and symmetry. Between two inputs a and b , the average rate of change is $(f(b) - f(a)) \div (b - a)$: output change over input change, which equals the slope of the secant line.

Standards: F-IF.B.4, F-IF.B.6, F-IF.C.9 · **Level:** Grade 9 Algebra 1

Which features describe the shape of a function?

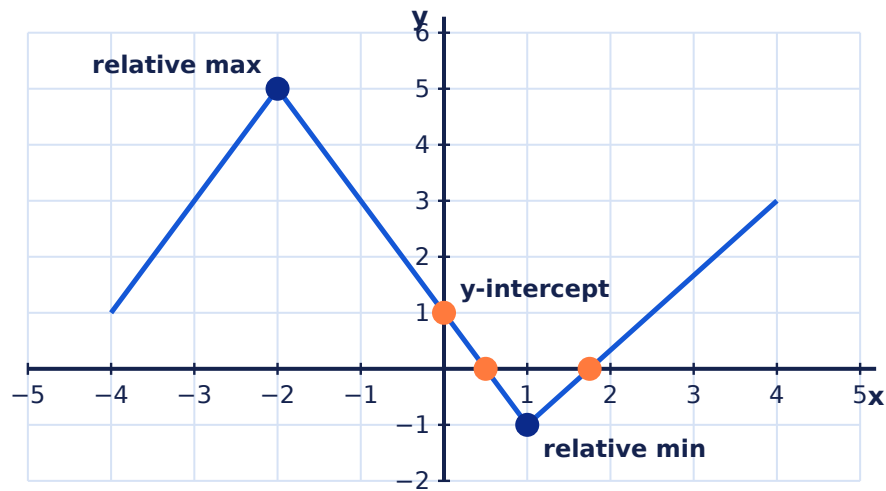
Each feature tells part of the story of a function, so reading them lets you describe a situation without any formula.



Intercepts and the maximum of $y = -(x - 1)^2 + 4$.

Vocabulary

- **Intercepts:** where the graph meets an axis. The y-intercept has $x = 0$ and an x-intercept (a zero) has $y = 0$.
- **Increasing / decreasing:** the outputs rise or fall as you move right.
- **Relative maximum / minimum:** a point higher or lower than all nearby points.
- **Positive / negative:** intervals where the graph is above or below the x-axis.



Increasing on $(-4, -2)$ and $(1, 4)$; decreasing on $(-2, 1)$.

Feature	How to find it	In the graph above
Increasing	graph rises left to right	$-4 < x < -2$ and $1 < x < 4$
Decreasing	graph falls left to right	$-2 < x < 1$
Relative maximum	peak between rising and falling	$(-2, 5)$

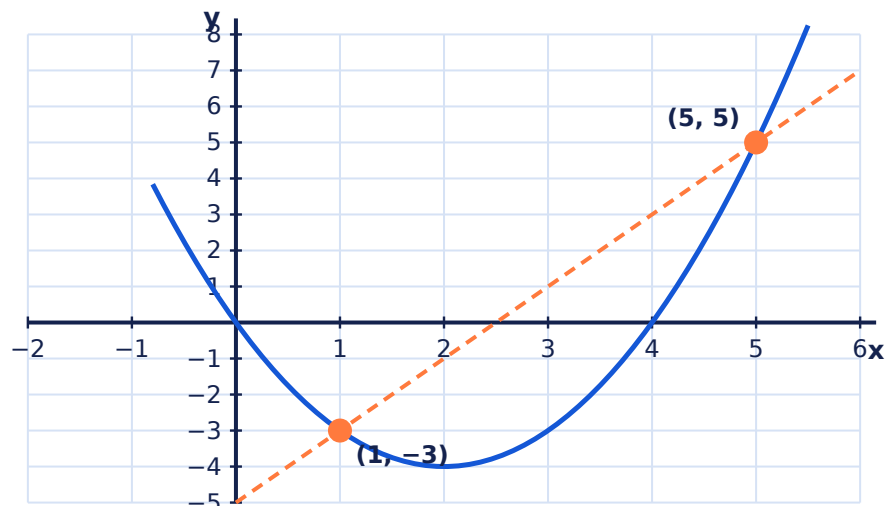
What is end behavior and symmetry?

End behavior tells what the outputs do as x grows very large in either direction. A line with positive slope rises to the right and falls to the left, while a parabola opening down falls at both ends. A parabola is **symmetric** about a vertical line through its vertex, so points at equal distances left and right of that line share the same height.

How is the average rate of change calculated?

Between inputs a and b , divide how much the output moved by how much the input moved.

$$\text{average rate of change} = \frac{f(b) - f(a)}{(b - a)}$$



The slope of the dashed secant is the average rate of change from $x = 1$ to $x = 5$.

Example: $f(x) = x^2 - 4x$ between $x = 1$ and $x = 5$

1. $f(1) = 1 - 4 = -3$ and $f(5) = 25 - 20 = 5$.
2. Rate = $(5 - (-3)) \div (5 - 1) = 8 \div 4 = 2$.

Common mistake

Mixing the order: subtract outputs and inputs in the same order. The units of the rate are output units per input unit, such as miles per hour.

Mathbot's tip

To compare a table, a graph and an equation, find the same feature from each, such as the rate of change or the y-intercept, in the same units.

Key points to remember

- A y-intercept has $x = 0$; an x-intercept (zero) has $y = 0$.
- A function is increasing where its graph rises as x increases.
- A relative maximum is higher than all nearby points.
- Average rate of change = $(f(b) - f(a)) \div (b - a)$, measured in output units per input unit.
- Compare functions by matching the same feature across table, graph and equation.

Key Features of Graphs and Rates of Change: frequently asked questions

How do you find the y-intercept from an equation?

Substitute $x = 0$ and compute the output. For $f(x) = 3x^2 - 5x + 7$, $f(0) = 7$, so the y-intercept is $(0, 7)$.

What is the difference between a relative and an absolute maximum?

A relative maximum is higher than the points right around it, but the function may reach even higher values elsewhere. An absolute maximum is the highest value on the whole graph.

What does a negative average rate of change mean?

The outputs decrease on average as the input increases. If distance from home goes from 12 miles to 6 miles in 3 hours, the rate is -2 miles per hour.

How can you tell if a graph is symmetric?

Look for a line that works like a mirror. A parabola is symmetric about the vertical line through its

vertex, so $f(h + d)$ equals $f(h - d)$ for any distance d .

How do you compare functions given in different forms?

Rewrite each in the same way, then compare one feature. For example, find the slope and y-intercept of a line from an equation and from a table, then say which is larger.