

# Key Features of Graphs and Rates of Change

Grade 9 – Algebra 1 math · Worksheet Answer Key · Standards: F-IF.B.4, F-IF.B.6, F-IF.C.9

Each answer key shows the points read from the graph or the substitution used in the rate formula. Compare the order of subtraction and the units in your own work, since the sign and units of a rate carry the meaning.

**Standards:** F-IF.B.4, F-IF.B.6, F-IF.C.9 · **Level:** Grade 9 Algebra 1

## Problem 1 Intercepts of a line

y-intercept:  $x = 0$  gives  $y = -6$ . x-intercept:  $0 = 2x - 6$  gives  $x = 3$ .

**(3, 0) and (0, -6)**

## Problem 2 Intercepts with a negative slope

$x = 0$ :  $y = 12$ .  $y = 0$ :  $3x = 12$ , so  $x = 4$ .

**(4, 0) and (0, 12)**

## Problem 3 Increasing or decreasing

The slope is  $-4$ , which is negative, so outputs fall as  $x$  increases.

**Decreasing**

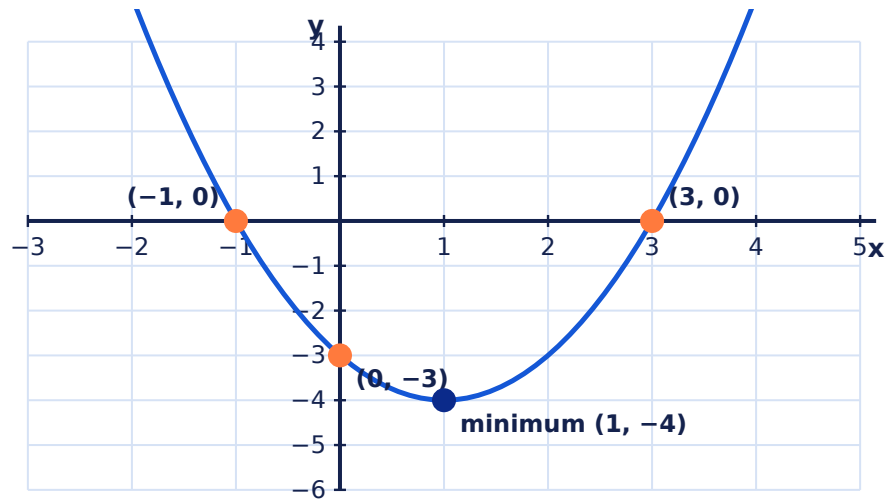
## Problem 4 Rate from a table

Outputs grow, so it is increasing. Rate =  $(28 - 16) \div (6 - 2) = 12 \div 4 = 3$ .

**Increasing; average rate of change 3**

## Problem 5 Reading a parabola

The graph crosses the x-axis at  $-1$  and  $3$ , crosses the y-axis at  $-3$  and its lowest point is  $(1, -4)$ .



*Intercepts and minimum are marked.*

**x-intercepts  $-1$  and  $3$ ; y-intercept  $-3$ ; minimum  $(1, -4)$**

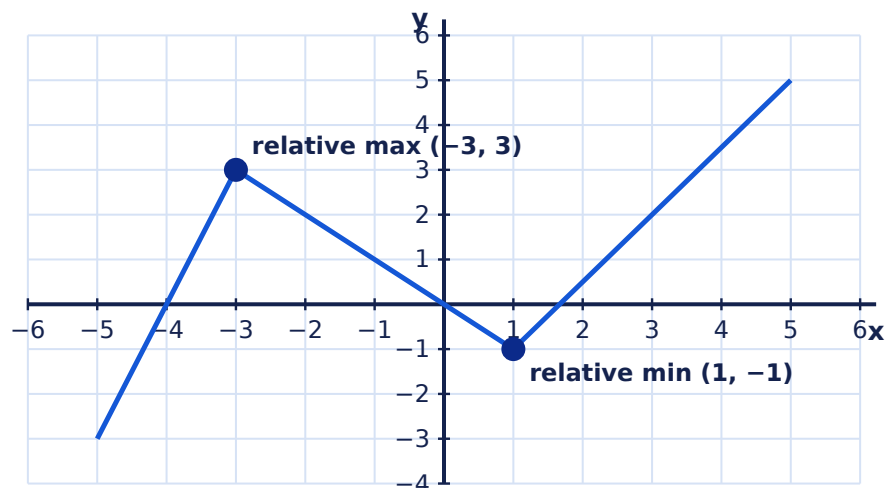
### Problem 6 Positive and negative intervals

The graph is above the x-axis to the left of  $-1$  and to the right of  $3$ , and below it between  $-1$  and  $3$ .

**Positive:  $x < -1$  or  $x > 3$ . Negative:  $-1 < x < 3$ .**

### Problem 7 Increasing, decreasing, extremes

The graph rises from  $x = -5$  to  $-3$ , falls from  $-3$  to  $1$ , and rises from  $1$  to  $5$ .



*Rises, falls, then rises again.*

**Increasing:  $-5 < x < -3$  and  $1 < x < 5$ . Decreasing:  $-3 < x < 1$ . Relative max  $(-3, 3)$ ; relative min  $(1, -1)$ .**

### Problem 8 Rate of change of a quadratic

$$f(2) = 4 - 12 + 5 = -3. \quad f(6) = 36 - 36 + 5 = 5.$$

$$\text{Rate} = (5 - (-3)) \div (6 - 2) = 8 \div 4 = 2.$$

**2**

### Problem 9 Rate with a negative input

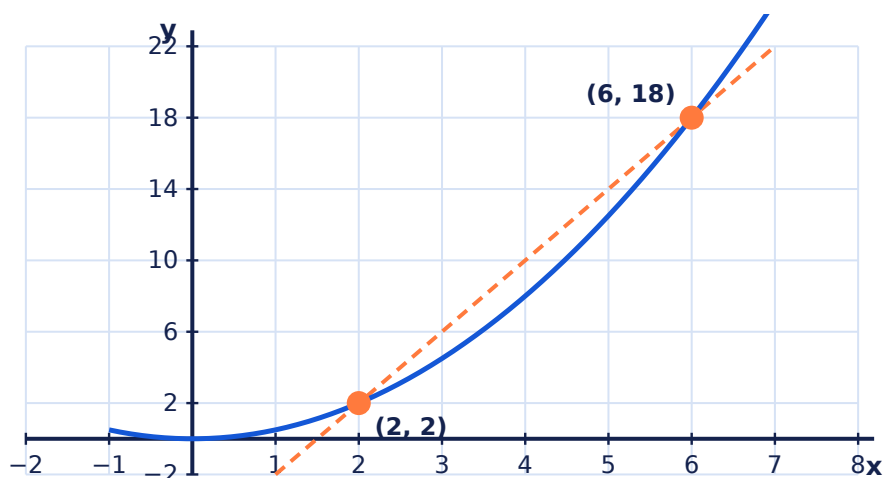
$$f(-4) = 16 - 11 = 5. \quad f(3) = 9 - 11 = -2.$$

$$\text{Rate} = (-2 - 5) \div (3 - (-4)) = -7 \div 7 = -1.$$

**-1**

### Problem 10 Rate from a graph

Points (2, 2) and (6, 18). Slope =  $(18 - 2) \div (6 - 2) = 16 \div 4 = 4$ .



*Slope of the secant = 4.*

**4**

### Problem 11 End behavior

(a) A line with negative slope: as  $x \rightarrow \infty$ ,  $f(x) \rightarrow -\infty$ , and as  $x \rightarrow -\infty$ ,  $f(x) \rightarrow +\infty$ .

(b) A parabola opening down:  $g(x) \rightarrow -\infty$  at both ends.

**(a) falls to the right, rises to the left (b) falls at both ends**

### Problem 12 Symmetry of a parabola

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The point 0 is 3 units left of the axis, so the mirror point is 3 units right:  $x = 6$ .

Check:  $f(0) = 9 - 2 = 7$  and  $f(6) = 9 - 2 = 7$ .

**(6, 7)**

### Problem 13 Compare an equation and a table

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$f$  has rate 4.  $g$  increases by 5 each step, and  $g(0) = 1$ , so  $g(x) = 5x + 1$ .

**$g$  has the greater rate (5 versus 4);  $g(x) = 5x + 1$**

### Problem 14 Balloon flight

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Increasing 0 to 10 min, constant 10 to 20 min, decreasing 20 to 35 min. The greatest altitude is 600 ft.

From 0 to 10:  $(600 - 0) \div 10 = 60$  ft/min. From 20 to 35:  $(0 - 600) \div 15 = -40$  ft/min.

**Max 600 ft; +60 ft/min and -40 ft/min**

### Problem 15 Bike trip table

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0 to 2:  $30 \div 2 = 15$  mph. 2 to 4:  $(60 - 30) \div 2 = 15$  mph. 1 to 3:  $(44 - 14) \div 2 = 15$  mph.

0 to 1:  $14 \div 1 = 14$  mph, which differs from 15.

**15, 15, 15 and 14 mph: not constant**

### Problem 16 Stock price

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0 to 4:  $(60 - 48) \div 4 = 3$  dollars per week (rising).

4 to 10:  $(42 - 60) \div 6 = -3$  dollars per week (falling).

0 to 10:  $(42 - 48) \div 10 = -0.6$  dollars per week overall.

**+3, -3 and -0.6 dollars per week**

### Problem 17 Two phone plans

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Plan B increases by 8 per 100 minutes, so  $C = 20 + 0.08m$  (check:  $20 + 8 = 28$ ). A's rate 0.12 is

greater than B's 0.08.

At 500 minutes:  $A = 18 + 60 = 78$  and  $B = 20 + 40 = 60$ .

Equal when  $18 + 0.12m = 20 + 0.08m$ , so  $0.04m = 2$  and  $m = 50$ .

**Plan A grows faster; \$78 versus \$60; equal at 50 minutes**

### Problem 18 Fish tank

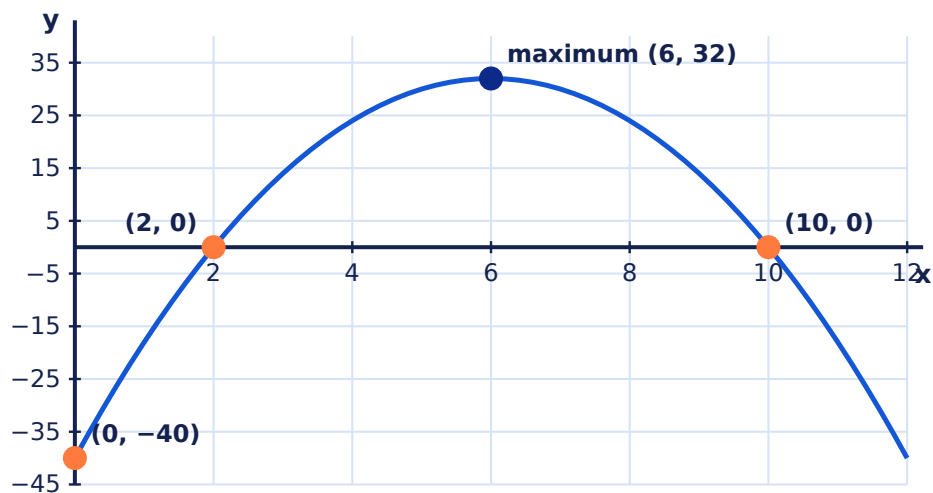
Decreasing from 0 to 6 min, constant (10 in) from 6 to 10 min, increasing from 10 to 16 min.  
Lowest level: 10 inches.

0 to 6:  $(10 - 40) \div 6 = -5$  in/min. 10 to 16:  $(40 - 10) \div 6 = 5$  in/min.

**Min 10 in; -5 in/min and +5 in/min**

### Problem 19 Cupcake profit

Zeros:  $x = 2$  and  $x = 10$  (break-even points). Maximum:  $P(6) = -72 + 144 - 40 = 32$ , so \$3,200 at 600 cupcakes. y-intercept:  $P(0) = -40$ , a loss of \$4,000 before any sale. Positive for  $2 < x < 10$ .



*Break-even points and the maximum profit.*

Average rate from 2 to 6:  $(32 - 0) \div 4 = 8$ .

**Zeros 2 and 10; max 32 at  $x = 6$ ; y-intercept -40; positive for  $2 < x < 10$**

### Problem 20 Compare two functions

$f$  has its maximum at  $(3, 9)$  and is positive for  $0 < x < 6$ , a width of 6.

g: vertex at  $x = 2$ ,  $g(2) = -4 + 8 + 8 = 12$ , so maximum 12. Its zeros are  $x = 2 \pm \sqrt{12} \approx -1.46$  and 5.46, a width  $\approx 6.93$ .

**g has the larger maximum (12 versus 9) and the wider positive interval (about 6.93 versus 6)**