

Solving, Graphing and Modeling with Inequalities

Grade 9 – Algebra 1 math · Lesson · Standards: A-REI.B.3, A-CED.A.1, A-CED.A.3

Quick answer: To solve a linear inequality, use the same steps as for an equation, but reverse the inequality sign whenever you multiply or divide both sides by a negative number. Graph the solution on a number line with an open or closed circle, and write it in interval notation such as $[-4, \infty)$.

Standards: A-REI.B.3, A-CED.A.1, A-CED.A.3 · **Level:** Grade 9 Algebra 1

How do you solve a linear inequality?

An inequality uses $<$, $>$, $\leq$ or $\geq$ instead of an equals sign, and its solution is a whole set of numbers. You solve it like an equation, with one new rule.

The sign-flip rule

Multiplying or dividing both sides by a **negative** number reverses the inequality sign. Adding or subtracting never changes it.

Method: solve $5 - 2x \leq 13$

1. Subtract 5 from both sides: $-2x \leq 8$.
2. Divide both sides by -2 and reverse the sign: $x \geq -4$.
3. Check a value: $x = 0$ gives $5 \leq 13$, which is true, and $0 \geq -4$ is true as well.
4. Graph a closed circle at -4 with the line shaded to the right.

How do you graph a solution?

A circle marks the boundary. It is open when the boundary is not a solution ($<$ or $>$) and closed when it is ($\leq$ or $\geq$).



$x > 2$: open circle, shade right.

Inequality	Circle	Direction	Interval notation
$x > 3$	open	right	$(3, \infty)$
$x \leq -1$	closed	left	$(-\infty, -1]$
$-2 \leq x < 5$	closed, open	between	$[-2, 5)$

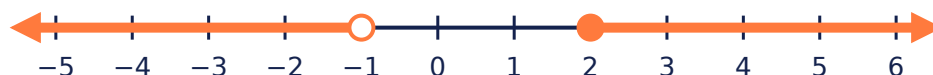
What are compound inequalities?

An **and** inequality such as $-3 < x \leq 4$ keeps the values that satisfy both parts, so its graph is one segment. Solve it by working on all three parts at once.



$-3 < x \leq 4$: a segment with one open and one closed end.

An **or** inequality keeps the values that satisfy at least one part, so its graph can be two separate rays.



$x < -1$ or $x \geq 2$: two separate rays.

Common mistake

Forgetting to reverse the sign when dividing by a negative number. Test one value from your answer in the original inequality.

How do you model a constraint?

Words such as "at most" ($\leq$), "at least" ($\geq$), "fewer than" ($<$) and "between" (compound) translate into inequality symbols. Define the variable first, then decide whether it must be a whole number.

Mathbot's tip

Pick a test value on the shaded side and plug it in. If the statement is true, the shading is right.

Key points to remember

- Reverse the sign only when multiplying or dividing by a negative number.
- Open circles and parentheses mean the boundary is excluded; closed circles and brackets include it.
- An **and**-inequality graphs as one segment; an **or**-inequality can graph as two rays.

- At most means $\leq$, at least means $\geq$, and between usually means a compound inequality.
- Test a value from the shaded region in the original inequality.

Solving, Graphing and Modeling with Inequalities: frequently asked questions

Why does the inequality sign flip?

Multiplying by a negative number reverses the order of numbers. For example, $2 < 5$ but $-2 > -5$. So $-2x \leq 8$ becomes $x \geq -4$ after dividing by -2 .

What is the difference between () and [] in interval notation?

Parentheses leave out the endpoint and brackets include it. Infinity always gets a parenthesis because it is not a number you can reach. So $x \geq 2$ is $[2, \infty)$ and $x > 2$ is $(2, \infty)$.

How do you solve a compound inequality such as $-7 < 2x - 1 \leq 9$?

Do the same operation to all three parts: add 1 to get $-6 < 2x \leq 10$, then divide by 2 to get $-3 < x \leq 5$. The graph is one segment.

When should a solution be a whole number?

When the variable counts items such as tickets or bars, only whole numbers make sense. Solve the inequality first, then keep the whole numbers that fit and round in the direction the constraint allows.

How can you check an inequality solution?

Choose one value inside the solution set and one outside it, and test both in the original inequality. The first should make it true and the second false.