

Combining and Multiplying Polynomials

Grade 9 – Algebra 1 math · Lesson · Standards: A-APR.A.1, A-SSE.A.2

Quick answer: A polynomial is a sum of terms made of numbers times whole-number powers of a variable. Add or subtract polynomials by combining like terms, and multiply them by distributing every term. Special products such as $(a + b)^2 = a^2 + 2ab + b^2$ shorten the work. The result is always another polynomial.

Standards: A-APR.A.1, A-SSE.A.2 · **Level:** Grade 9 Algebra 1

What is a polynomial and how do you describe it?

A polynomial is a sum of terms, and each term is a number times a variable raised to a whole-number power. The expression $3x + 8$ can model a taxi fare, and $0.5x^2 + 4x$ can model the height of a launched rocket.

Vocabulary

- **Coefficient:** the number multiplying the variable part.
- **Degree:** the highest power of the variable.
- **Standard form:** terms written from highest power to lowest.
- **Monomial, binomial, trinomial:** one, two or three terms.

degree = 3 (highest power)

$$4x^3 - 7x^2 + x - 9$$



leading term
coefficient 4



term: x
(coefficient 1)



constant term
-9

Parts of a polynomial in standard form.

How do you add and subtract polynomials?

Combine like terms, which have the same variable and the same power. To subtract, change the sign of every term in the second polynomial first.

Worked example

$$(6x^2 - x + 2) - (2x^2 + 4x - 5) = 6x^2 - x + 2 - 2x^2 - 4x + 5 = \mathbf{4x^2 - 5x + 7}.$$

Common mistake

Changing only the first sign of the second polynomial. The minus sign reaches every term inside the parentheses.

How do you multiply polynomials?

Method: distribute every term

1. Multiply each term of the first polynomial by each term of the second.
2. Use FOIL (first, outer, inner, last) for two binomials, or an area model for larger ones.
3. Combine like terms and write the answer in standard form.

	x	6
x	x^2	$6x$
3	$3x$	18

$$(x + 3)(x + 6) = x^2 + 6x + 3x + 18 = x^2 + 9x + 18$$

Which special products should you know?

Special products

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

	x	5
x	x^2	$5x$
5	$5x$	25

$$(x + 5)^2 = x^2 + 5x + 5x + 25 = x^2 + 10x + 25$$

These patterns also speed up arithmetic: $29 \times 31 = (30 - 1)(30 + 1) = 900 - 1 = 899$.

Why do polynomials stay polynomials?

Adding, subtracting or multiplying two polynomials always gives another polynomial. Mathematicians call this closure. Division can break it: $1 \div x$ is not a polynomial.

Mathbot's tip

Squaring a binomial always makes three terms. If your answer has only two, the middle term went missing.

Key points to remember

- The degree is the highest power; standard form lists powers from high to low.
- Like terms have the same variable raised to the same power.
- When subtracting, change the sign of every term in the second polynomial.
- $(a + b)^2 = a^2 + 2ab + b^2$ and $(a + b)(a - b) = a^2 - b^2$.
- Polynomials are closed under addition, subtraction and multiplication.

Combining and Multiplying Polynomials: frequently asked questions

What is the degree of a polynomial?

It is the highest exponent on the variable after like terms are combined. The polynomial $5x^4 - 2x + 9$ has degree 4.

What are like terms?

Like terms have the same variable raised to the same power, such as $7x^2$ and $-3x^2$. Only like terms can be combined by adding their coefficients.

Is $(x + 4)^2$ equal to $x^2 + 16$?

No. Squaring means multiplying the binomial by itself, so $(x + 4)^2 = x^2 + 8x + 16$. The middle term $8x$ comes from the two cross products.

What is the difference between FOIL and the box method?

Both multiply every term by every term. FOIL is a memory aid for two binomials only, while the box method organizes any product in a grid.

What does closure mean for polynomials?

Closure means that adding, subtracting or multiplying polynomials always produces a polynomial. Dividing polynomials does not always do so.