

Graphing Parabolas and Key Features

Grade 9 – Algebra 1 math · Lesson · Standards: F-IF.C.7a, F-IF.C.8a, F-IF.B.4, A-SSE.B.3b

Quick answer: A quadratic function graphs as a parabola. Its vertex is the lowest point when the leading coefficient is positive and the highest point when it is negative. The mirror line is $x = -\frac{b}{2a}$. Standard form reveals the y-intercept, vertex form reveals the vertex, and factored form reveals the zeros.

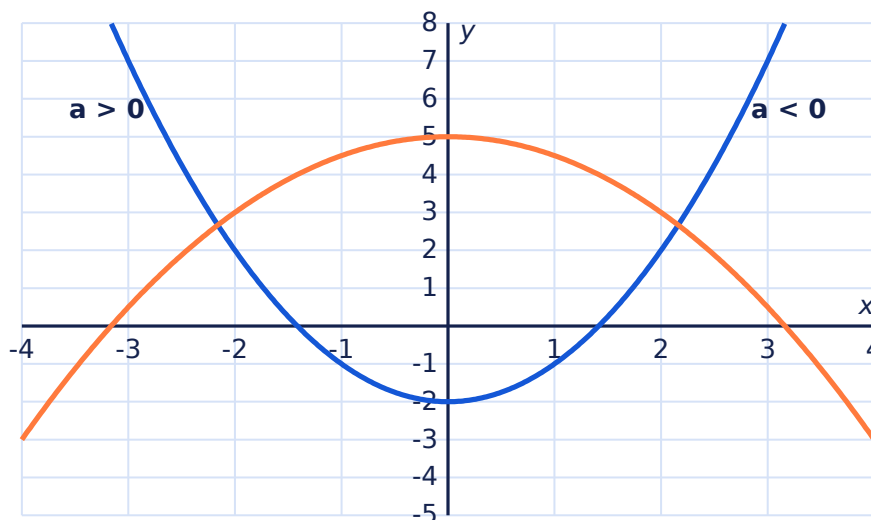
Standards: F-IF.C.7a, F-IF.C.8a, F-IF.B.4, A-SSE.B.3b · **Level:** Grade 9 Algebra 1

What does the graph of a quadratic function look like?

When $a \neq 0$, the rule $f(x) = ax^2 + bx + c$ is quadratic. Its picture is a parabola, the U-shaped path of a basketball shot, or the area of a rectangle as one side stretches.

Key features

- **Vertex:** the turning point, a minimum when $a > 0$ and a maximum when $a < 0$.
- **Axis of symmetry:** a mirror line, $x = h$, that cuts the parabola into two matching halves.
- **Intercepts:** the points where the curve meets the x-axis or the y-axis.

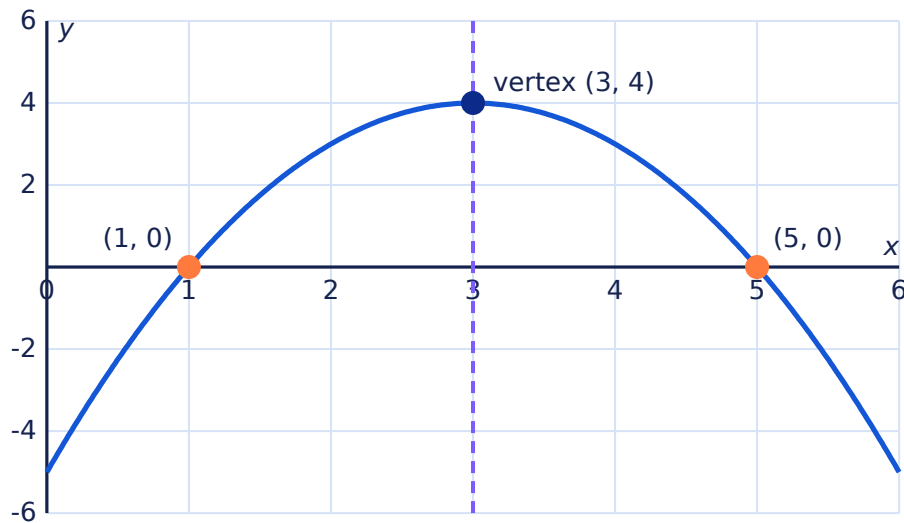


$a > 0$ opens up (minimum); $a < 0$ opens down (maximum).

Which three forms should you know?

- Standard form $y = ax^2 + bx + c$ gives the point $(0, c)$ at once, and the mirror line $x = -\frac{b}{2a}$.
- Vertex form $y = a(x - h)^2 + k$ shows the turning point (h, k) .

- Factored form $y = a(x - p)(x - q)$ shows crossings of the x -axis at p and q .



$y = -(x - 3)^2 + 4$: the vertex $(h, k) = (3, 4)$.

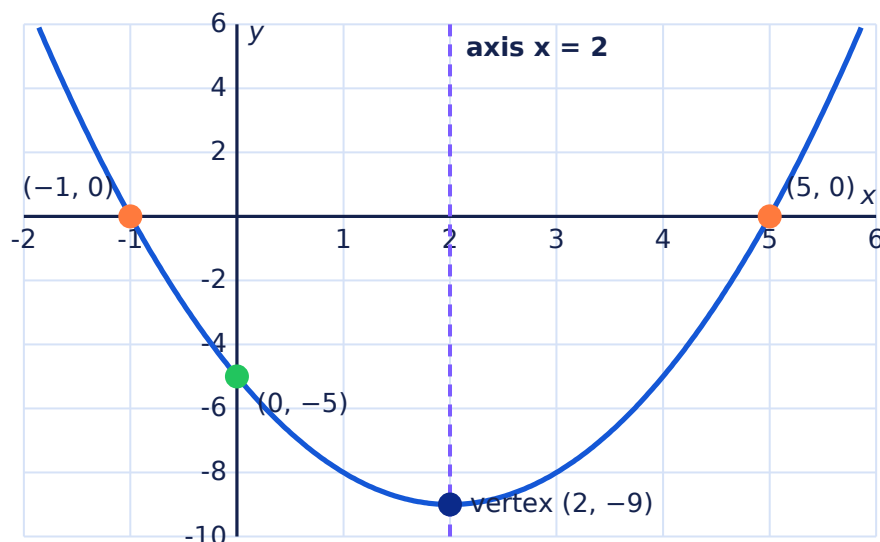
How do you find the vertex from standard form?

Method

1. Compute $x = -b \div (2a)$, the x -coordinate of the turning point.
2. Put that x back into the rule to get y .
3. Plot (x, y) , then add the intercepts to sketch the curve.

Worked example

$y = x^2 - 4x - 5$. Here $x = 4 \div 2 = 2$ and $y = 4 - 8 - 5 = -9$, so the turning point is $(2, -9)$. Factoring gives $(x - 5)(x + 1)$, so the curve crosses the x -axis at 5 and -1 and the y -axis at -5 .



How does completing the square give the vertex form?

Add and subtract the square of half the x-coefficient. For $y = x^2 + 6x + 5$, half of 6 is 3, so $y = (x^2 + 6x + 9) - 9 + 5 = (x + 3)^2 - 4$. The vertex is $(-3, -4)$.

Common mistake

In $(x + 3)^2$ the vertex has $x = -3$, not 3. The sign inside the parentheses is the opposite of h .

Mathbot's tip

The vertex sits halfway between the two zeros, so average the zeros to get the axis.

Key points to remember

- The parabola opens up if $a > 0$ and down if $a < 0$.
- Axis of symmetry in standard form: $x = -\frac{b}{2a}$.
- Vertex form $y = a(x - h)^2 + k$ has vertex (h, k) .
- Factored form $y = a(x - p)(x - q)$ has zeros p and q .
- Completing the square turns standard form into vertex form.

Graphing Parabolas and Key Features: frequently asked questions

How do you find the vertex of a parabola?

In standard form, compute $x = -\frac{b}{2a}$, then substitute it into the function to get y . In vertex form $y = a(x - h)^2 + k$, the vertex is simply (h, k) .

What is the axis of symmetry?

It is the vertical line through the vertex that splits the parabola into two mirror-image halves. Its equation is $x = h$, where h is the x-coordinate of the vertex.

How do you know whether a parabola has a maximum or a minimum?

Look at the sign of a . If a is positive the graph opens up and the vertex is a minimum. If a is negative it opens down and the vertex is a maximum.

What does completing the square do?

It rewrites $x^2 + bx + c$ as $(x + \frac{b}{2})^2 + k$, a perfect square plus a constant. This reveals the vertex and makes some equations easy to solve.

How can you find the zeros of a quadratic function?

Set the function equal to 0 and factor, if possible. In factored form $y = a(x - p)(x - q)$ the zeros p and q can be read directly from the factors.