

Graphing Parabolas and Key Features

Grade 9 – Algebra 1 math · Worksheet Answer Key · Standards: F-IF.C.7a, F-IF.C.8a, F-IF.B.4, A-SSE.B.3b

Each answer shows how the vertex, intercepts or equation was found, with the final result in bold. If your vertex differs, recompute the axis with $x = -\frac{b}{2a}$ and check the sign. Answers read from graphs should match the marked points.

Standards: F-IF.C.7a, F-IF.C.8a, F-IF.B.4, A-SSE.B.3b · **Level:** Grade 9 Algebra 1

Problem 1 Read the Vertex Form, Opening Up

$a = 2 > 0$, so it opens up. In $a(x - h)^2 + k$ the vertex is (3, 5), the axis is $x = 3$ and the minimum value is 5.

Opens up; vertex (3, 5); axis $x = 3$; minimum 5

Problem 2 Read the Vertex Form, Opening Down

$a = -1 < 0$, so it opens down. Write $x + 4$ as $x - (-4)$: the vertex is $(-4, -2)$. The maximum value is -2 .

Opens down; vertex $(-4, -2)$; maximum -2

Problem 3 Axis from Standard Form

Mirror line: $x = 18 \div 2 = 9$. Then $y = 81 - 162 + 50 = -31$.

Mirror line $x = 9$; turning point (9, -31)

Problem 4 Complete a Table of Values

$x = 0$: 5. $x = 1$: $1 - 6 + 5 = 0$. $x = 2$: $4 - 12 + 5 = -3$. $x = 3$: $9 - 18 + 5 = -4$. $x = 4$: -3 . $x = 5$: 0. $x = 6$: 5.

The lowest value is at $x = 3$, and the values mirror around it.

Vertex (3, -4); y-intercept 5

Problem 5 Read the Factored Form

Zeros: 2 and -6 . The axis is halfway: $(2 + (-6)) \div 2 = -2$. At $x = -2$: $(-4)(4) = -16$.

Zeros 2 and -6 ; axis $x = -2$; vertex $(-2, -16)$

Problem 6 Read a Parabola Graph

The highest point is $(2, 5)$, so the vertex is $(2, 5)$ and the axis is $x = 2$. The graph crosses the y -axis at $(0, 1)$.

Vertex $(2, 5)$; axis $x = 2$; maximum 5; y -intercept 1

Problem 7 Vertex Form to Standard Form

$(x - 1)^2 = x^2 - 2x + 1$. Multiply by 3: $3x^2 - 6x + 3$. Subtract 4.

$y = 3x^2 - 6x - 1$

Problem 8 Vertex of a Downward Parabola

$x = -12 \div (2 \cdot (-2)) = 3$. $y = -2(9) + 36 - 7 = 11$.

Vertex $(3, 11)$; maximum value 11

Problem 9 Complete the Square, Easy Case

Half of 10 is 5: $y = (x^2 + 10x + 25) - 25 + 16 = (x + 5)^2 - 9$. Vertex $(-5, -9)$.

Zeros: $x^2 + 10x + 16 = (x + 2)(x + 8)$, so $x = -2$ or $x = -8$. They are symmetric around $x = -5$.

$y = (x + 5)^2 - 9$; vertex $(-5, -9)$; zeros -2 and -8

Problem 10 Completing the Square, No Zeros

Half of -8 is -4 : $y = (x^2 - 8x + 16) - 16 + 19 = (x - 4)^2 + 3$.

The vertex $(4, 3)$ is the minimum and lies above the x -axis, and the parabola opens up, so $y \geq 3$ for all x .

$y = (x - 4)^2 + 3$; no x -intercepts

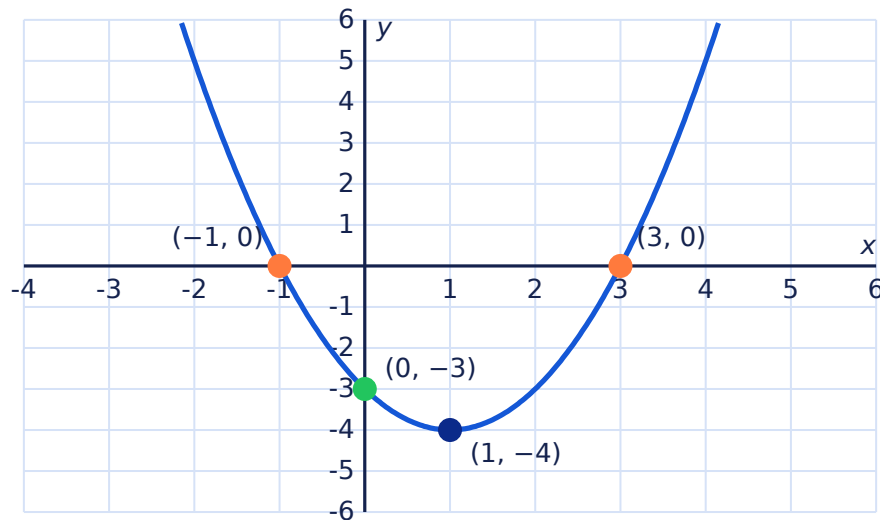
Problem 11 Completing the Square, $a = 2$

Factor 2 from the first two terms: $2(x^2 - 6x) + 13$. Add 9 inside: $2(x^2 - 6x + 9) - 2 \cdot 9 + 13$.
 $= 2(x - 3)^2 - 18 + 13 = 2(x - 3)^2 - 5$.

$y = 2(x - 3)^2 - 5$; vertex $(3, -5)$

Problem 12 Graph from Zeros and Vertex

Factor: $(x - 3)(x + 1)$, so the zeros are 3 and -1 . Axis $x = 1$, and $y = 1 - 2 - 3 = -4$. The y -intercept is -3 .



Zeros -1 and 3 ; vertex $(1, -4)$; y -intercept -3

Problem 13 Vertex from Factored Form

Zeros: -1 and 5 . Axis: $(-1 + 5) \div 2 = 2$. At $x = 2$: $-2(3)(-3) = 18$.

Since $a = -2 < 0$, the parabola opens down.

Vertex $(2, 18)$; maximum value 18

Problem 14 Compare Three Parabolas

A larger $|a|$ makes the parabola narrower. $|3| > |1| > |-0.5|$.

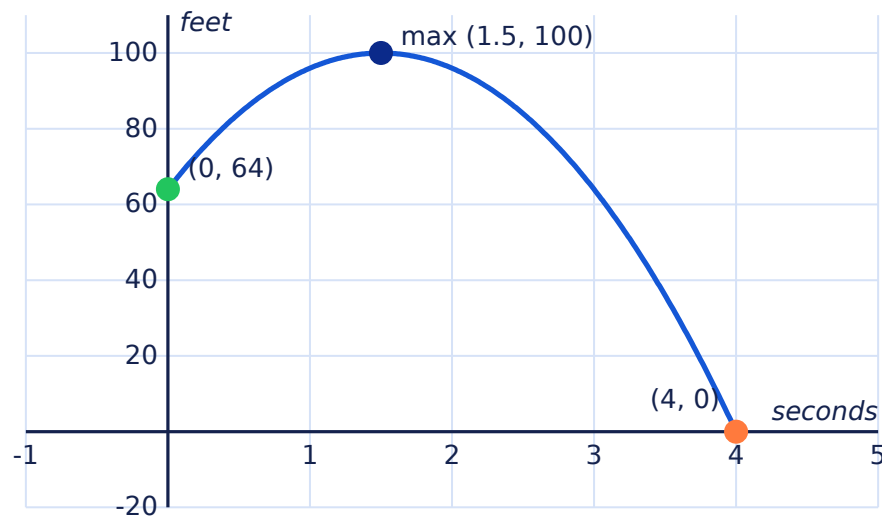
The sign of a sets the direction: only $-0.5x^2$ opens down.

Narrowest: $y = 3x^2$; opens down and is widest: $y = -0.5x^2$

Problem 15 Ball Launched from a Roof

Starting height: $h(0) = 64$. Maximum at $t = -48 \div (2 \cdot (-16)) = 1.5$, so $h(1.5) = -36 + 72 + 64 = 100$.

Ground: $-16t^2 + 48t + 64 = -16(t - 4)(t + 1) = 0$, so $t = 4$ (reject -1).



Key points of the ball path.

64 ft; 100 ft at 1.5 s; lands at 4 s

Problem 16 Largest Rectangular Pen

Zeros: $w = 0$ and $w = 48$, so the vertex is at $w = 24$. $A(24) = 24 \cdot 24 = 576$.

The best pen is a square, 24 ft by 24 ft.

Width 24 ft; maximum area 576 square feet

Problem 17 Best Ticket Price

Zeros: $p = 0$ and $250 - 5p = 0$, so $p = 50$. The vertex is halfway at $p = 25$.

$R(25) = 25(250 - 125) = 25 \times 125 = 3125$.

Price \$25; maximum revenue \$3,125; revenue is 0 at \$0 and \$50

Problem 18 Equation from Vertex and a Point

Start with $y = a(x + 1)^2 + 6$. Substitute $(2, -3)$: $-3 = a(3)^2 + 6$, so $9a = -9$ and $a = -1$.

Standard: $-(x^2 + 2x + 1) + 6 = -x^2 - 2x + 5$.

$$y = -(x + 1)^2 + 6 = -x^2 - 2x + 5$$

Problem 19 Equation from Zeros and a Point

Start with $y = a(x + 2)(x - 4)$. At $(0, -16)$: $-16 = a(2)(-4) = -8a$, so $a = 2$.

Check the standard form $2x^2 - 4x - 16$: at $x = 0$ it equals -16 ✓.

$$y = 2(x + 2)(x - 4)$$

Problem 20 Compare Two Quadratic Functions

$f(x) = (x - 3)^2 + 2$, so the minimum is 2 at $x = 3$.

The table of g is symmetric around $x = 1$, where the lowest value is 1. So the minimum of g is 1 at $x = 1$.

g has the smaller minimum (1 at $x = 1$, against 2 at $x = 3$ for f)