

Working with Exponents, Roots and Fractional Powers

Grade 9 – Algebra 1 math · Lesson · Standards: N-RN.A.1, N-RN.A.2

Quick answer: Integer exponent rules combine powers of the same base: add exponents when multiplying, subtract when dividing and multiply for a power of a power. A rational exponent m/n means the n th root raised to the m th power, so $8^{(2/3)} = (\sqrt[3]{8})^2 = 4$. Simplify a square root by pulling out its largest perfect square factor.

Standards: N-RN.A.1, N-RN.A.2 · **Level:** Grade 9 Algebra 1

Which exponent rules should you know?

An exponent counts repeated multiplication. When the base is the same, the rules turn multiplication of powers into addition of exponents, and so on.

Product: $x^a \cdot x^b = x^{a+b}$

Quotient: $x^a \div x^b = x^{a-b}$

Power: $(x^a)^b = x^{ab}$

Zero: $x^0 = 1$ Negative: $x^{-n} = 1/x^n$

The five integer-exponent rules.

Rule	Example
product	$x^3 \cdot x^5 = x^8$
quotient	$x^9 \div x^4 = x^5$
power of a power	$(x^2)^6 = x^{12}$
negative exponent	$3^{-2} = \frac{1}{9}$

Example: simplify $(2x^3)^2 \cdot x^{-4}$

1. Raise each factor to the power: $(2x^3)^2 = 2^2 \cdot x^6 = 4x^6$.
2. Multiply by x^{-4} and add exponents: $4x^6 \cdot x^{-4} = 4x^2$.

Zero and negative exponents follow from the quotient rule. Since $x^3 \div x^3 = x^0$ and also equals 1, we define $x^0 = 1$ for nonzero x . In the same way, $x^2 \div x^5 = x^{-3} = 1/x^3$, so a negative exponent moves a factor across the fraction bar.

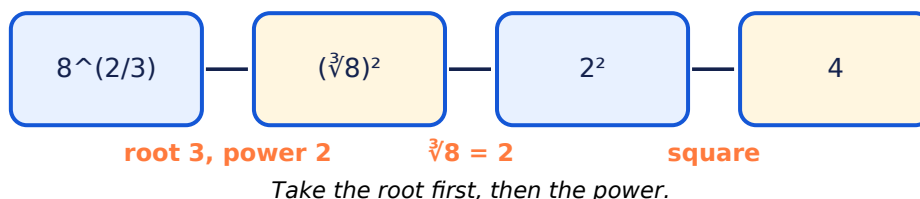
Common mistake

Adding exponents when the bases are added or multiplied differently. Only powers of the *same* base combine, and in $(2x)^3$ the 2 is cubed too, giving $8x^3$.

What does a rational exponent mean?

A fractional exponent combines a root and a power: the denominator is the root and the numerator is the power.

$$x^{m/n} = (\sqrt[n]{x})^m$$

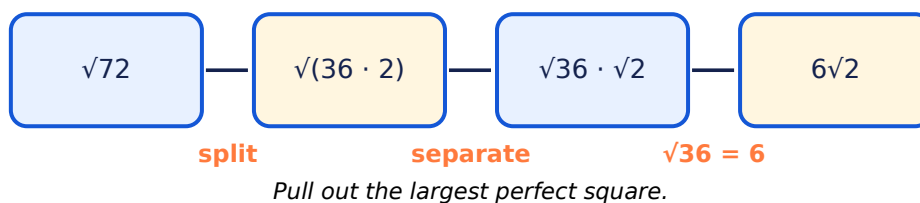


Method: rewrite and evaluate $16^{3/4}$

1. Read the denominator 4 as a fourth root: $\sqrt[4]{16} = 2$.
2. Read the numerator 3 as a power: 2^3 .
3. Compute: $16^{3/4} = 8$.

How do you simplify a square root?

Factor out the largest perfect square, then use $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.



So $\sqrt{72} = 6\sqrt{2}$, which is about 8.49. The number $\sqrt{72}$ is irrational, so the decimal never ends.

Perfect squares	4	9	16	25	36	49	64
Square roots	2	3	4	5	6	7	8

To simplify any square root, divide the number by these perfect squares until one fits. For $\sqrt{50}$, the square 25 fits: $\sqrt{50} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$. When no perfect square greater than 1 divides the number, as with $\sqrt{35}$, the root is already in simplest form.

Mathbot's tip

Know the perfect squares up to 144 and the perfect cubes 8, 27, 64 and 125. They let you spot roots instantly.

Key points to remember

- Same base: $x^a \cdot x^b = x^{a+b}$ and $x^a \div x^b = x^{a-b}$.
- $(x^a)^b = x^{ab}$, and $x^0 = 1$ for nonzero x .
- A negative exponent means a reciprocal: $x^{-n} = 1/x^n$.
- $x^{m/n}$ is the n th root of x raised to the m th power.
- Simplify $\sqrt{72}$ by writing it as $\sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$.

Working with Exponents, Roots and Fractional Powers: frequently asked questions

Why does x^0 equal 1?

By the quotient rule, $x^3 \div x^3 = x^0$, but any nonzero number divided by itself is 1. So x^0 must equal 1 for every nonzero x .

What does a negative exponent mean?

It means the reciprocal. For example, $3^{-2} = 1/3^2 = 1/9$. A negative exponent never makes the number negative.

How do you read $8^{(2/3)}$?

The denominator 3 is a cube root and the numerator 2 is a square. So $8^{(2/3)} = (\sqrt[3]{8})^2 = 2^2 = 4$. Taking the root first keeps the numbers small.

How do you simplify $\sqrt{72}$?

Find the largest perfect square dividing 72, which is 36. Then $\sqrt{72} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$, about 8.49.

Is the sum of two square roots a single root?

No. $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$, but $\sqrt{25} = 5$. Roots multiply and divide nicely, $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$, but addition does not combine inside the root.