

Solving Quadratic Equations Four Ways

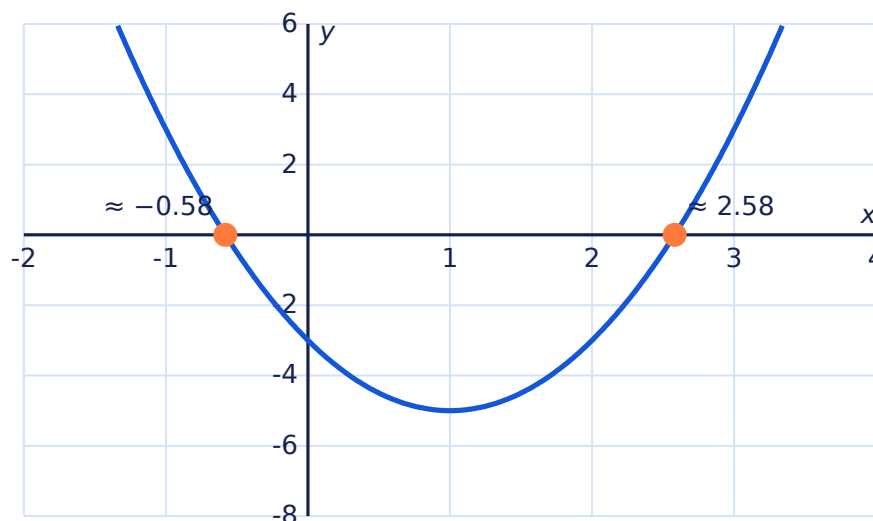
Grade 9 – Algebra 1 math · Lesson · Standards: A-REI.B.4a, A-REI.B.4b, A-APR.B.3, A-CED.A.1

Quick answer: A quadratic equation can be solved by taking square roots, factoring with the zero-product property, completing the square or using the quadratic formula. Each solution is an x-intercept of the parabola. The discriminant $b^2 - 4ac$ tells how many real solutions exist: positive means two, zero means one, and negative means none.

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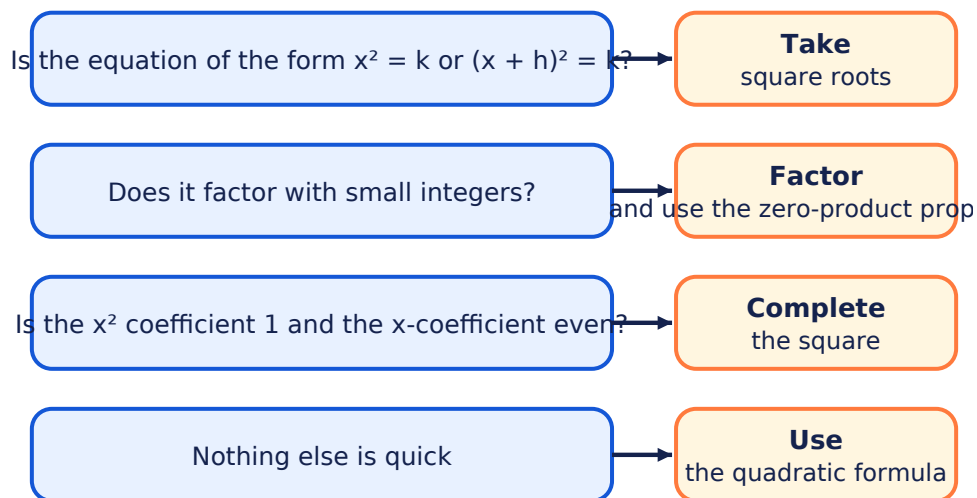
What does it mean to solve a quadratic equation?

To solve $ax^2 + bx + c = 0$ is to find every x that makes the left side zero. On the graph of $y = ax^2 + bx + c$ these solutions are the x-intercepts, also called roots or zeros. A quadratic can have two, one or no real solutions.



The solutions of $2x^2 - 4x - 3 = 0$ are the x-intercepts.

Which method should you choose?



Pick the quickest method that works.

Method	Best for	Example
Square roots	no middle term	$x^2 = 49$ gives $x = \pm 7$
Factoring	small integer factors	$(x - 3)(x + 8) = 0$
Completing the square	$a = 1$ and even b	$(x + 4)^2 = 21$
Quadratic formula	any quadratic	a, b, c substituted

Quadratic formula

For $ax^2 + bx + c = 0$ with $a \neq 0$: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

How does completing the square work?

Method

1. Move the constant to the right side.
2. Add (half of b)² to both sides.
3. Write the left side as a perfect square.
4. Take square roots, then solve for x .

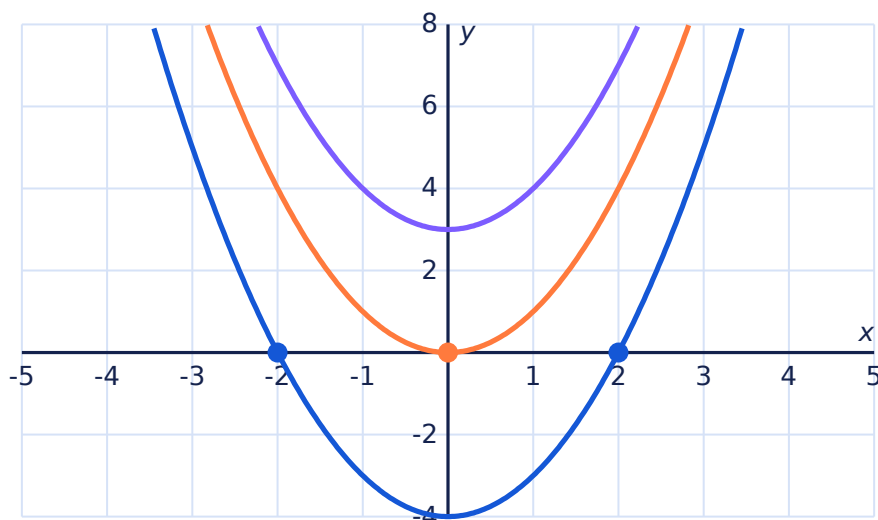
Worked example

$x^2 + 6x - 5 = 0$ becomes $x^2 + 6x = 5$. Add 9: $(x + 3)^2 = 14$, so $x = -3 \pm \sqrt{14}$, or about 0.74 and -6.74.

Running these same steps on $ax^2 + bx + c = 0$ produces the quadratic formula, which is why the formula works for every quadratic.

How does the discriminant predict the answers?

The expression $b^2 - 4ac$ under the square root is the discriminant. Positive means two real solutions, zero means exactly one, and negative means none because a negative number has no real square root.



Two crossings, one touch, no crossing: the discriminant decides.

Common mistake

Writing only the + case of $\pm$. Keep both signs, then simplify each answer separately.

Mathbot's tip

Move everything to one side so the equation equals 0 before you factor or use the formula.

Key points to remember

- Solutions of $ax^2 + bx + c = 0$ are the x-intercepts of $y = ax^2 + bx + c$.
- Quadratic formula: $x = (-b \pm \sqrt{b^2 - 4ac}) \div 2a$.
- Discriminant $b^2 - 4ac$: positive gives 2, zero gives 1, negative gives 0 real solutions.
- Zero-product property: if $m \cdot n = 0$ then $m = 0$ or $n = 0$.
- Reject answers that make no sense in a word problem, such as negative lengths.

Solving Quadratic Equations Four Ways: frequently asked questions

How many solutions can a quadratic equation have?

It can have two real solutions, exactly one real solution, or no real solution. The graph crosses the x-axis twice, touches it once, or stays entirely above or below it.

When is factoring the best method?

Factoring is quickest when the quadratic splits into small integer factors, such as $x^2 + 5x + 6$. If it does not factor easily, completing the square or the quadratic formula always works.

Why must the equation equal zero before factoring?

The zero-product property only works when a product equals 0. Saying that $(x - 2)(x + 3) = 10$ does not mean $x - 2 = 10$, so move all terms to one side first.

What does the discriminant tell you?

The discriminant $b^2 - 4ac$ shows the number of real solutions without solving. A value above 0 gives two solutions, exactly 0 gives one, and below 0 gives none.

Where does the quadratic formula come from?

It comes from completing the square on $ax^2 + bx + c = 0$. Doing the steps with letters instead of numbers gives a formula that works for any a , b and c .