

Solving Quadratic Equations Four Ways

Grade 9 – Algebra 1 math · Worksheet Answer Key · Standards: A-REI.B.4a, A-REI.B.4b, A-APR.B.3, A-CED.A.1

Each answer shows the chosen method, the key algebra steps and the solutions in bold. When an answer involves a square root, both the exact value and a rounded decimal are given. Substitute a solution back into the equation to check it.

Standards: A-REI.B.4a, A-REI.B.4b, A-APR.B.3, A-CED.A.1 · **Level:** Grade 9 Algebra 1

Problem 1 Square Roots, Plain Case

Take both square roots: $x = \pm\sqrt{81}$.

$x = 9$ or $x = -9$

Problem 2 Square Roots After Dividing

Divide by 3: $x^2 = 25$. Then $x = \pm 5$.

$x = 5$ or $x = -5$

Problem 3 Square Roots with a Shifted Square

Take square roots: $x - 4 = 7$ or $x - 4 = -7$.

Add 4: 11 or -3 .

$x = 11$ or $x = -3$

Problem 4 Zero-Product Property

Either $x + 6 = 0$ or $x - 2 = 0$.

$x = -6$ or $x = 2$

Problem 5 Factor and Solve

Factor: $(x + 2)(x + 5) = 0$.

$x = -2$ or $x = -5$

Problem 6 No Constant Term

Factor out x : $x(x - 9) = 0$.

Either $x = 0$ or $x - 9 = 0$. Do not divide by x , because that would lose the solution 0.

$x = 0$ or $x = 9$

Problem 7 Factor a Trinomial Equation

Product -30 and sum -7 : -10 and 3 . So $(x - 10)(x + 3) = 0$.

$x = 10$ or $x = -3$

Problem 8 Factor with a Leading 2

$ac = -20$ with sum 1: 5 and -4 . Split: $2x^2 - 4x + 5x - 10 = 2x(x - 2) + 5(x - 2) = (2x + 5)(x - 2)$.

$2x + 5 = 0$ gives $x = -\frac{5}{2}$.

$x = -\frac{5}{2}$ or $x = 2$

Problem 9 Irrational Square Roots

$x + 2 = \pm\sqrt{18} = \pm 3\sqrt{2}$. So $x = -2 \pm 3\sqrt{2}$.

$3\sqrt{2} \approx 4.243$, so $x \approx 2.24$ or $x \approx -6.24$.

$x = -2 \pm 3\sqrt{2}$ (about 2.24 and -6.24)

Problem 10 Complete the Square, Positive Middle

$x^2 + 8x = 5$. Half of 8 is 4 , so add 16 : $x^2 + 8x + 16 = 21$.

$(x + 4)^2 = 21$, so $x = -4 \pm \sqrt{21} \approx -4 \pm 4.583$.

$x \approx 0.58$ or $x \approx -8.58$

Problem 11 Complete the Square, Negative Middle

$x^2 - 6x = -2$. Add 9 : $(x - 3)^2 = 7$.

$x = 3 \pm \sqrt{7} \approx 3 \pm 2.646$.

$$x \approx 5.65 \text{ or } x \approx 0.35$$

Problem 12 Complete the Square, Leading 4

Divide by 4: $x^2 + 2x = \frac{1}{4}$. Add 1: $(x + 1)^2 = \frac{5}{4}$.

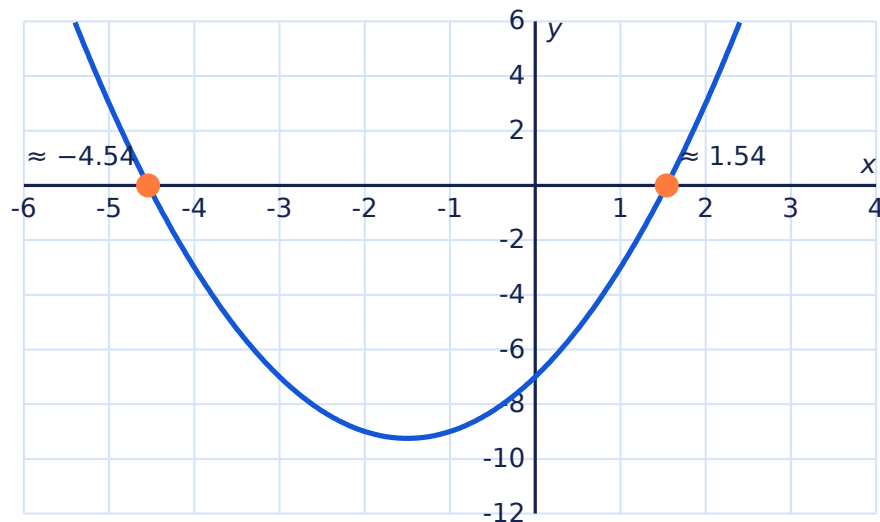
$x + 1 = \pm \sqrt{\frac{5}{4}}$, so $x = -1 \pm \sqrt{\frac{5}{4}} \approx -1 \pm 1.118$.

$$x = -1 \pm \sqrt{\frac{5}{4}} \text{ (about 0.12 and -2.12)}$$

Problem 13 Quadratic Formula, First Try

$a = 1$, $b = 3$, $c = -7$. The discriminant is $9 + 28 = 37$.

$$x = \frac{-3 \pm \sqrt{37}}{2} \approx \frac{-3 \pm 6.083}{2}.$$



The two solutions from the quadratic formula match the graph.

$$x \approx 1.54 \text{ or } x \approx -4.54$$

Problem 14 Count the Solutions

- a. $36 - 36 = 0$: one real solution ($x = 3$).
- b. $9 - 40 = -31$: no real solution.
- c. $1 + 4 = 5$: two real solutions.

(a) one; (b) none; (c) two

Problem 15 Formula with $a = 3$

$a = 3, b = -5, c = -4$. Discriminant: $25 + 48 = 73$.

$$x = \frac{(5 \pm \sqrt{73})}{6} \approx \frac{(5 \pm 8.544)}{6}.$$

$x \approx 2.26$ or $x \approx -0.59$

Problem 16 Pick the Fast Method

Square roots are quickest because the equation has a squared binomial. Divide by 4: $(x - 1)^2 = 25$.

$x - 1 = \pm 5$, so $x = 6$ or $x = -4$.

$x = 6$ or $x = -4$ (square roots)

Problem 17 Rectangular Plot

Let the width be w . Then $w(w + 3) = 154$, so $w^2 + 3w - 154 = 0$.

Factor: $(w + 14)(w - 11) = 0$. Width cannot be -14 .

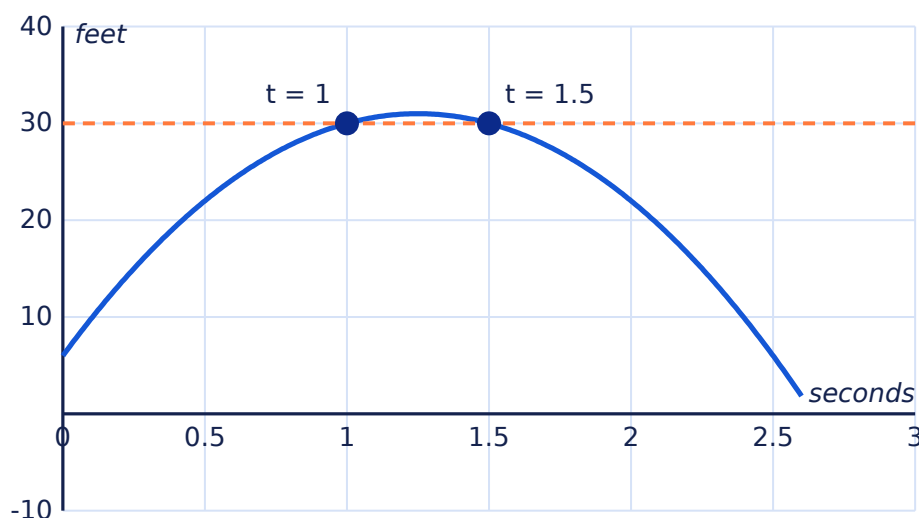
Width 11 ft, length 14 ft. Check: $11 \times 14 = 154$ ✓.

11 ft by 14 ft

Problem 18 Height Equal to 30 Feet

Set $-16t^2 + 40t + 6 = 30$, so $-16t^2 + 40t - 24 = 0$. Divide by -8 : $2t^2 - 5t + 3 = 0$.

Factor: $(2t - 3)(t - 1) = 0$, so $t = 1$ or $t = \frac{3}{2}$.



Times when the height equals 30 feet.

At 1 second (rising) and 1.5 seconds (falling)

Problem 19 Landing Time with the Formula

Set $-16t^2 + 30t + 8 = 0$ and multiply by -1 : $16t^2 - 30t - 8 = 0$, then divide by 2: $8t^2 - 15t - 4 = 0$.

Discriminant: $225 + 128 = 353$. $t = \frac{(15 \pm \sqrt{353})}{16} \approx \frac{(15 \pm 18.788)}{16}$.

The minus case is negative, so reject it. $t \approx 33.788 \div 16 \approx 2.11$.

About 2.11 seconds

Problem 20 Exactly One Solution

Exactly one solution needs discriminant 0: $36 - 4k = 0$, so $k = 9$.

None needs a negative discriminant: $36 - 4k < 0$, so $k > 9$.

$k = 9$ for one solution; $k > 9$ for none