

Graphing Inequalities and Systems with Shaded Regions

Grade 9 – Algebra 1 math · Worksheet Answer Key · Standards: A-REI.D.12, A-CED.A.3

Each answer key states the boundary line, whether it is solid or dashed, the test point and the shading. For systems, a test point in the overlap confirms the answer.

Standards: A-REI.D.12, A-CED.A.3 · **Level:** Grade 9 Algebra 1

Problem 1 Test two points

$(3, 9)$: $9 \geq 2(3) + 1 = 7$ is true. $(1, 2)$: $2 \geq 2(1) + 1 = 3$ is false.

$(3, 9)$ is a solution; $(1, 2)$ is not.

Problem 2 Solid or dashed

(a) Strict inequality: dashed. (b) $\geq$ includes the line: solid. (c) $\leq$ includes the line: solid.

(a) dashed (b) solid (c) solid

Problem 3 Which side to shade

$0 > -6(0) + 14 = 14$ is false, so the origin is not in the solution. Shade the side away from the origin, above the line.

Shade above the dashed line.

Problem 4 Write from a graph

The dashed line passes through $(0, 2)$ with slope 1: $y = x + 2$. The shading is below the line and the line is dashed.

$y < x + 2$

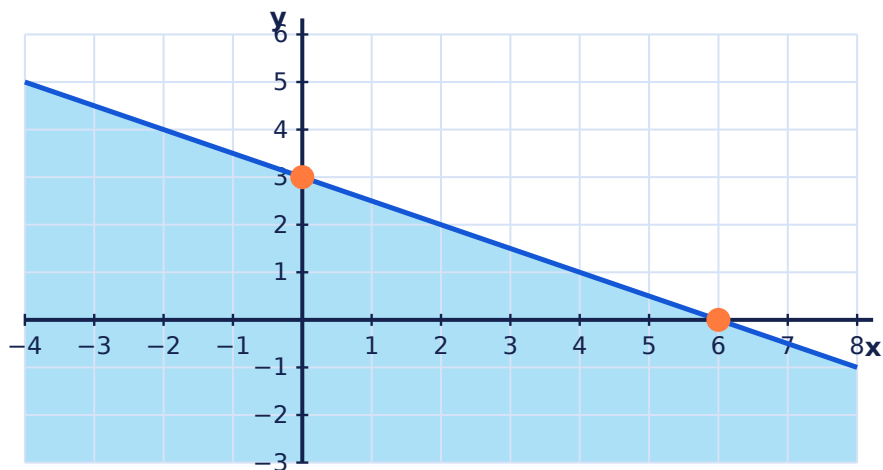
Problem 5 Check a point in a system

First: $3 < -2 + 6 = 4$ is true. Second: $3 \geq -(-2) = 2$ is true.

Yes, $(-2, 3)$ is a solution.

Problem 6 Graph a solid half-plane

The boundary has y-intercept 3 and slope $-1/2$, through $(0, 3)$ and $(6, 0)$. It is solid. Test $(0, 0)$: $0 \leq 3$ is true, so shade below.

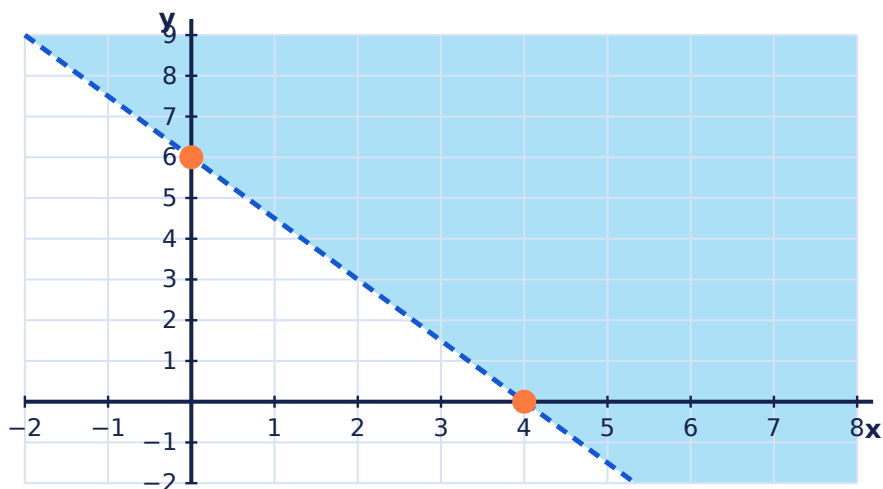


$$y \leq -\frac{1}{2}x + 3: \text{solid, shade below.}$$

Solid line, shade below

Problem 7 Graph from standard form

The boundary $3x + 2y = 12$ crosses the axes at $(4, 0)$ and $(0, 6)$, and is dashed. Test $(0, 0)$: $0 > 12$ is false, so shade the side away from the origin, above the line.

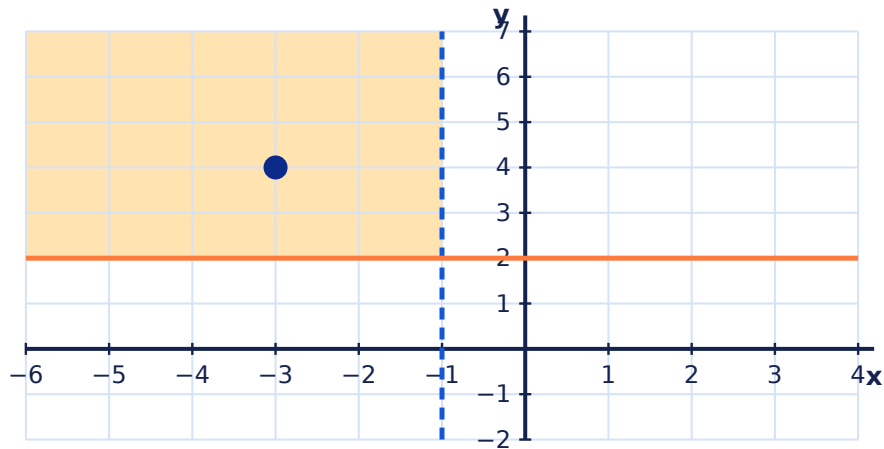


$$3x + 2y > 12: \text{dashed, shade above.}$$

Dashed line, shade above

Problem 8 Vertical and horizontal boundaries

$x = -1$ is a dashed vertical line with shading to the left. $y = 2$ is a solid horizontal line with shading above. The overlap is the upper-left region.



$x < -1$ and $y \geq 2$: the upper-left region.

Test $(-3, 4)$: $-3 < -1$ and $4 \geq 2$ are both true.

Upper-left region

Problem 9 Solve for y first

$-2y \leq -4x + 10$. Divide by -2 and reverse the sign: $y \geq 2x - 5$.

The boundary $y = 2x - 5$ is solid, with shading above.

$y \geq 2x - 5$; solid line, shade above

Problem 10 Write a system from a graph

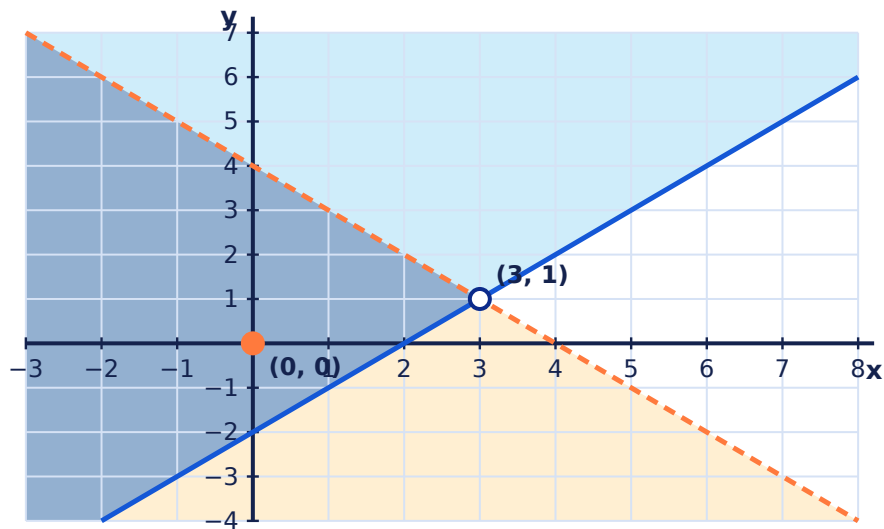
The dashed line $y = x$ has the shading above it: $y > x$. The solid horizontal line $y = 4$ has the shading below it: $y \leq 4$.

Test $(0, 2)$: $2 > 0$ and $2 \leq 4$ are both true.

$y > x$ and $y \leq 4$

Problem 11 Graph a system

The boundary $y = x - 2$ is solid with shading above; $y = -x + 4$ is dashed with shading below. They meet at $x - 2 = -x + 4$, so $x = 3$ and $y = 1$; this corner is not a solution because one line is dashed.



The overlap contains $(0, 0)$; the corner $(3, 1)$ is not included.

Test $(0, 0)$: $0 \geq -2$ and $0 < 4$ are true.

Yes, $(0, 0)$ is a solution.

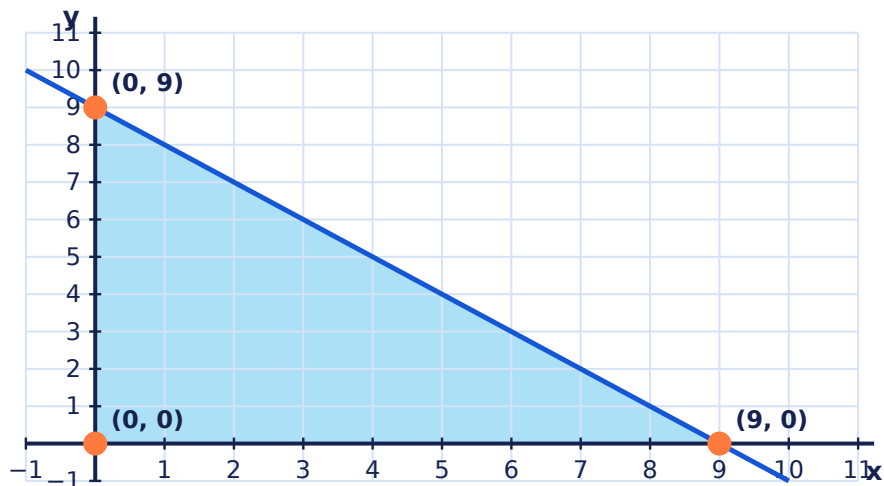
Problem 12 Test several points

$(4, 1)$: $9 \leq 10$ and $3 > 1$: yes. $(5, 3)$: $13 \leq 10$ is false. $(3, -2)$: $4 \leq 10$ and $5 > 1$: yes. $(6, 0)$: $12 \leq 10$ is false.

$(4, 1)$ and $(3, -2)$

Problem 13 Triangle region

The first two bound the first quadrant; $x + y = 9$ crosses the axes at $(9, 0)$ and $(0, 9)$. The solution is a triangle.



Triangle with vertices $(0, 0)$, $(9, 0)$, $(0, 9)$.

Corners $(0, 0)$, $(9, 0)$ and $(0, 9)$

Problem 14 No overlap

Both boundaries have slope 8 and are parallel. The first needs y above the line $y = 8x + 3$; the second needs y below the lower line $y = 8x - 7$. No point is both above the upper line and below the lower one.

Algebra: $8x + 3 \leq y \leq 8x - 7$ would require $3 \leq -7$, which is false.

No solution

Problem 15 Snack cart budget

$$2.5p + 2j \leq 30.$$

$(6, 7)$: $15 + 14 = 29 \leq 30$, allowed. $(8, 5)$: $20 + 10 = 30 \leq 30$, allowed (on the line). $(9, 6)$: $22.5 + 12 = 34.5$, not allowed.

The boundary line passes through $(12, 0)$ and $(0, 15)$.

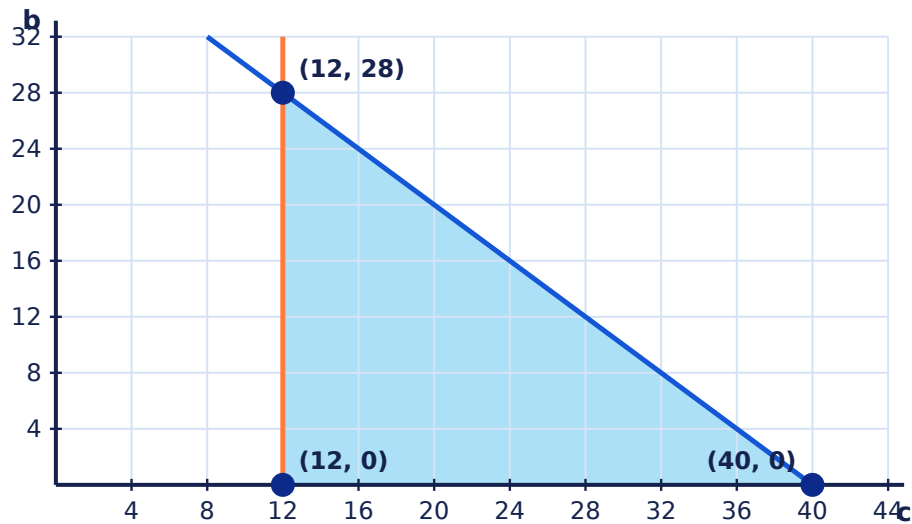
$2.5p + 2j \leq 30$; $(6, 7)$ and $(8, 5)$ work, $(9, 6)$ does not

Problem 16 Bake sale constraints

$$c + b \leq 40, c \geq 12, b \geq 0.$$

Corners: $c = 12$ with $b = 0$ gives $(12, 0)$; $b = 0$ with $c + b = 40$ gives $(40, 0)$; $c = 12$ with $c + b = 40$ gives $(12, 28)$.

$(15, 20)$: $35 \leq 40$ and $15 \geq 12$, feasible. $(10, 25)$: $10 < 12$, not feasible.



Corner points of the feasible region.

Corners (12, 0), (40, 0), (12, 28); (15, 20) yes, (10, 25) no

Problem 17 Shipping limits

With k kits and s sensors: $3k + 0.5s \leq 30$, $k \geq 2$, $s \geq 0$.

(5, 20): $15 + 10 = 25 \leq 30$ and $5 \geq 2$, allowed. (9, 10): $27 + 5 = 32$, too heavy.

With 2 kits: $6 + 0.5s \leq 30$, so $s \leq 48$.

(5, 20) yes; (9, 10) no; 48 sensors

Problem 18 Hiking food plan

$250b + 400p \geq 2,000$ (which is $5b + 8p \geq 40$) and $2b + 5p \leq 40$.

(4, 3): $1,000 + 1,200 = 2,200 \geq 2,000$ and $8 + 15 = 23 \leq 40$, feasible. (2, 2): $500 + 800 = 1,300 < 2,000$, not feasible. (10, 4): $2,500 + 1,600 \geq 2,000$ and $20 + 20 = 40 \leq 40$, feasible (on the boundary).

(4, 3) and (10, 4) are feasible; (2, 2) is not

Problem 19 Weekly earnings plan

$12b + 15t \geq 240$ and $b + t \leq 20$ (with $b, t \geq 0$).

(10, 8): $120 + 120 = 240 \geq 240$ and $18 \leq 20$, feasible. (15, 3): $180 + 45 = 225 < 240$, no. (4, 12): $48 + 180 = 228$, no. (5, 15): $60 + 225 = 285 \geq 240$ and $20 \leq 20$, feasible.

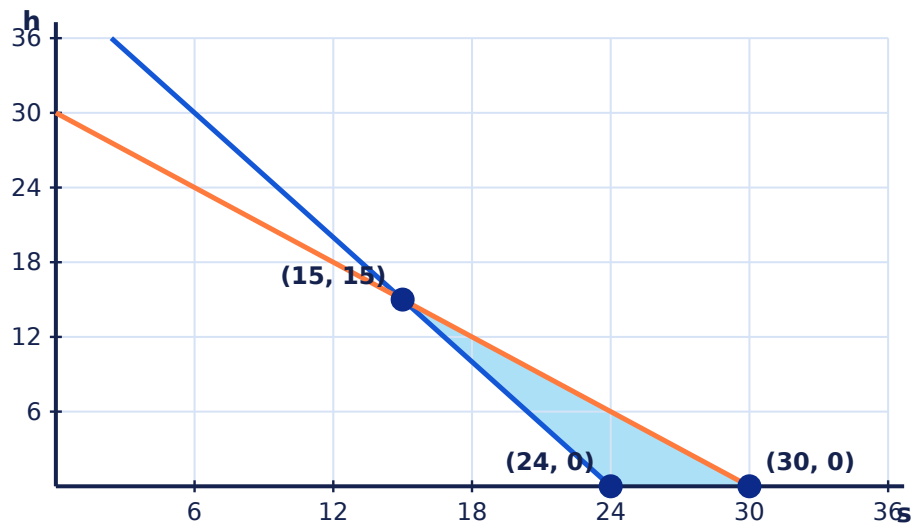
(10, 8) and (5, 15)

Problem 20 Shirts and hats

$$15s + 9h \geq 360 \text{ and } s + h \leq 30.$$

Meeting point: $h = 30 - s$ gives $15s + 9(30 - s) = 360$, so $6s = 90$, $s = 15$ and $h = 15$.

The feasible region is the triangle with corners $(24, 0)$, $(30, 0)$ and $(15, 15)$.



Shirts (s) and hats (h): the feasible triangle.

For $(20, 8)$: $300 + 72 = 372 \geq 360$ and $28 \leq 30$, so it works.

Boundaries meet at $(15, 15)$; 20 shirts and 8 hats work