

Real and Complex Numbers: Working with i

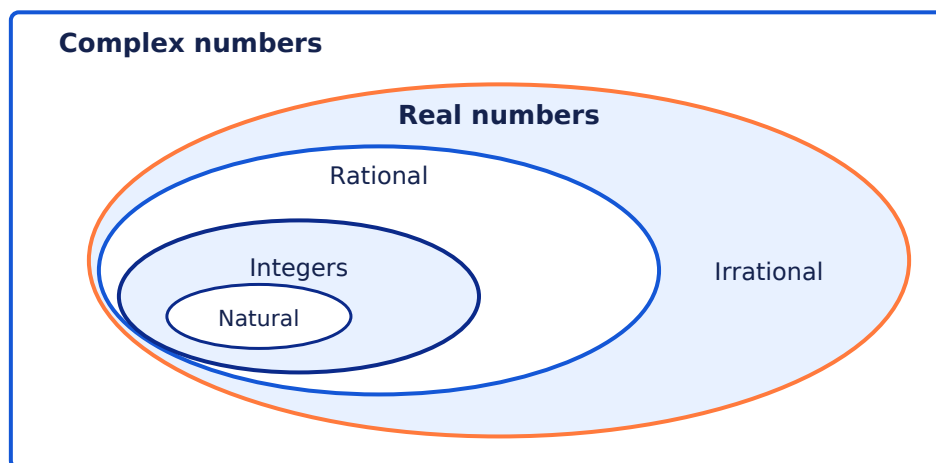
Grade 11 – Algebra 2 math · Lesson · Standards: N-RN.A.1, N-RN.A.2, N-CN.A.1, N-CN.A.2

Quick answer: A complex number has the form $a + bi$, where a and b are real numbers and i is the imaginary unit with $i^2 = -1$. Every real number is complex with $b = 0$. Complex numbers add, subtract, and multiply like binomials, and each i^2 becomes -1 .

Standards: N-RN.A.1, N-RN.A.2, N-CN.A.1, N-CN.A.2 · **Level:** Grade 11 Algebra 2

How do the number sets fit together?

Every number belongs to a nested family. **Natural numbers** are 1, 2, 3, ...; **integers** add 0 and the negatives; **rational numbers** are quotients of integers such as $-7/4$, and their decimals stop or repeat. **Irrational numbers** such as $\sqrt{17}$ and π have decimals that never stop and never repeat. Rational and irrational numbers together fill the number line: they are the **real numbers**.



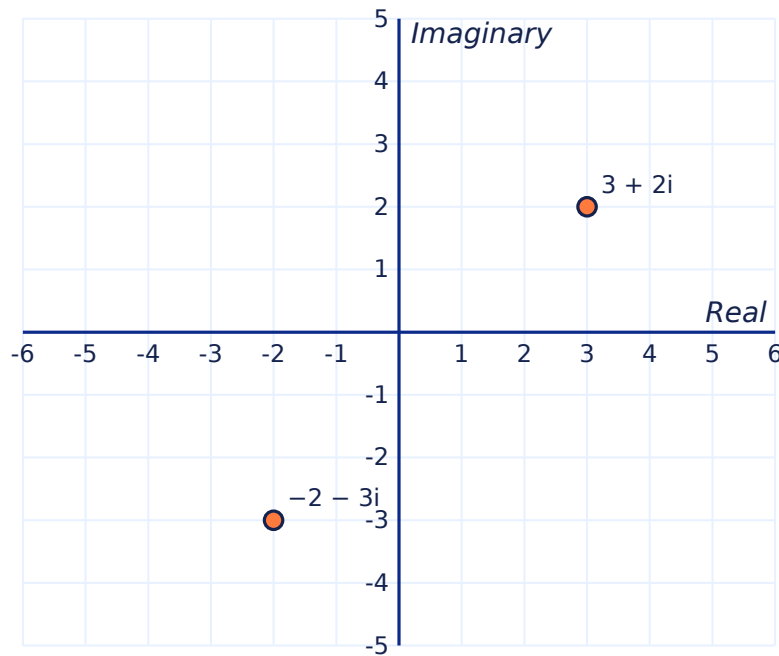
Each set sits inside the next one. The complex numbers contain every real number.

What is the imaginary unit?

Squaring a real number never gives a negative result, so a new number is defined to fill the gap.

Imaginary unit and complex numbers

The imaginary unit is $i = \sqrt{-1}$, so $i^2 = -1$. A complex number is any number $a + bi$ with a and b real. Here a is the real part and b is the imaginary part. For $n > 0$, $\sqrt{-n} = i\sqrt{n}$.



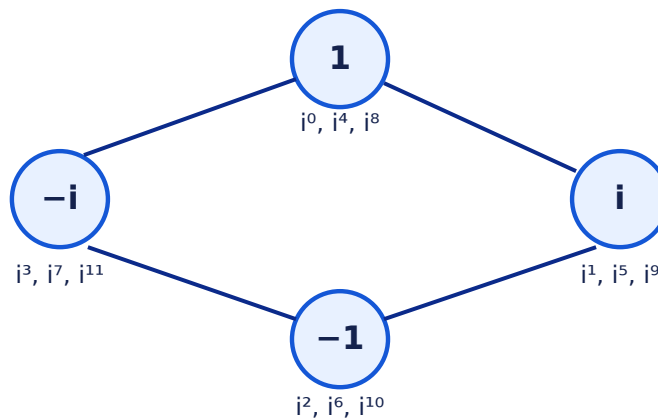
The complex plane: the real part is the horizontal position, the imaginary part is the vertical position.

How do you add, subtract, and multiply complex numbers?

Operation	Rule	Example
Add	Combine real parts and imaginary parts	$(2 + 5i) + (4 - 9i) = 6 - 4i$
Subtract	Distribute the minus sign, then combine	$(2 + 5i) - (4 - 9i) = -2 + 14i$
Multiply	Use FOIL, then replace i^2 with -1	$(1 + 3i)(2 + i) = 2 + i + 6i + 3i^2 = -1 + 7i$

How do you find a power of i ?

The powers of i repeat in a cycle of length 4: i , -1 , $-i$, 1 .



Divide the exponent by 4 and use the remainder to read the value from the cycle.

Method: evaluate i^n

1. Divide the exponent n by 4.

2. Keep the remainder 0, 1, 2, or 3.
3. Match it: 0 gives 1, 1 gives i , 2 gives -1 , 3 gives $-i$.

Worked example

Evaluate i^{62} . Since $62 = 4 \cdot 15 + 2$, the remainder is 2, so $i^{62} = i^2 = -1$.

Common mistake

The rule $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ fails for negative radicands. Write i first: $\sqrt{(-4)} \cdot \sqrt{(-9)} = 2i \cdot 3i = -6$, not 6.

Mathbot's tip

Pull out i before doing any other step. Once every root is written as a multiple of i , ordinary algebra finishes the job.

Key points to remember

- The imaginary unit satisfies $i^2 = -1$.
- A complex number $a + bi$ has real part a and imaginary part b .
- Real numbers are the complex numbers whose imaginary part is 0.
- Powers of i repeat every 4 steps: $i, -1, -i, 1$.
- Add and subtract by parts; multiply with FOIL and replace i^2 by -1 .

Real and Complex Numbers: Working with i : frequently asked questions

Why is the imaginary unit called imaginary?

The name dates from a time when negative square roots seemed useless. Today the numbers are as legitimate as any other and are used daily in electrical engineering and signal processing. The word is only a historical label.

Is every real number also a complex number?

Yes. A real number such as 7 can be written $7 + 0i$, so it is a complex number whose imaginary part is zero. The real numbers form a subset of the complex numbers.

Why does $\sqrt{(-4)} \cdot \sqrt{(-9)}$ equal -6 instead of 6 ?

The product rule for square roots only holds when the radicands are not negative. Rewrite each root first as $2i$ and $3i$, then multiply to get $6i^2 = -6$.

How do you evaluate a very large power of i ?

Divide the exponent by 4 and look only at the remainder. A remainder of 0, 1, 2, or 3 gives 1, i , -1 , or $-i$ respectively.

Where do complex numbers appear outside of math class?

Alternating-current circuits use them to track voltage, current, and impedance together. Computer graphics and sound processing also use complex arithmetic because multiplying by i turns a point by 90 degrees.