

Real and Complex Numbers: Working with i

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: N-RN.A.1, N-RN.A.2, N-CN.A.1, N-CN.A.2

Every solution below shows the key steps and the final answer in bold. Compare each method with the work on the worksheet, paying attention to where i^2 is replaced by -1 and to sign errors.

Standards: N-RN.A.1, N-RN.A.2, N-CN.A.1, N-CN.A.2 · **Level:** Grade 11 Algebra 2

Problem 1 Sorting numbers into sets

- a) $0.375 = \frac{3}{8}$: **rational, real**.
- b) $\sqrt{45} = 3\sqrt{5}$ is not a perfect square root: **irrational, real**.
- c) $-\sqrt{64} = -8$: **integer, rational, real** (not natural).
- d) $\frac{5}{9}$ is a quotient of integers: **rational, real**.
- e) $\sqrt{-25} = 5i$: **non-real (complex)**.

Problem 2 Simplifying negative roots

- a) $\sqrt{-81} = 9i$.
- b) $\sqrt{-44} = \sqrt{4} \cdot \sqrt{11} \cdot i = 2i\sqrt{11}$.
- c) $\sqrt{-72} = 6i\sqrt{2}$, so the opposite is $-6i\sqrt{2}$.

Problem 3 Reading a point on the complex plane

The point is 4 units right and 3 units down, so $Z = 4 - 3i$.

Real part 4, imaginary part -3 . The conjugate is $4 + 3i$.

Problem 4 Adding and subtracting

- a) Real: $6 + (-9) = -3$. Imaginary: $2 + 7 = 9$. **$-3 + 9i$**
- b) **$4 - 11i + 5 + 6i = 9 - 5i$** .

Problem 5 Identifying parts

- a) Real 9, imaginary -4 .
- b) Real 0, imaginary -7 .
- c) Real 12, imaginary 0.
- d) Real $\frac{1}{2}$, imaginary $\frac{3}{4}$.

(a) 9 and -4 ; (b) 0 and -7 ; (c) 12 and 0; (d) $\frac{1}{2}$ and $\frac{3}{4}$

Problem 6 Plotting and naming points

Read the horizontal position, then the vertical position.

$A = 5 - 2i$, $B = -3 + 4i$, $C = -1 - 3i$, $D = 4i$

Problem 7 Multiplying binomials

FOIL: $20 - 30i + 12i - 18i^2$.

Replace i^2 by -1 : $20 - 18i + 18 = \mathbf{38 - 18i}$.

Problem 8 Products with conjugates

- a) $-21i^2 = 21$.
- b) $4 + 2i - 2i - i^2 = 4 + 1 = 5$.
- c) $25 - 36i^2 = 25 + 36 = 61$.

(a) 21; (b) 5; (c) 61

Problem 9 Powers of an imaginary number

- a) $9i^2 = -9$.
- b) $32i^5 = 32i$ (since $i^5 = i$).
- c) $-64i^3 = -64(-i) = 64i$.

(a) -9 ; (b) $32i$; (c) $64i$

Problem 10 Product of negative roots

a) $3i \cdot 4i = 12i^2 = -12$.

b) $i\sqrt{5} \cdot 2i\sqrt{5} = 10i^2 = -10$.

(a) -12; (b) -10

Problem 11 Conjugate product

a) Conjugate $8 + 3i$. Product $64 - 9i^2 = 64 + 9 = 73$.

b) Conjugate $-5i$. Product $-25i^2 = 25$.

(a) $8 + 3i$ and 73; (b) $-5i$ and 25

Problem 12 Equal complex numbers

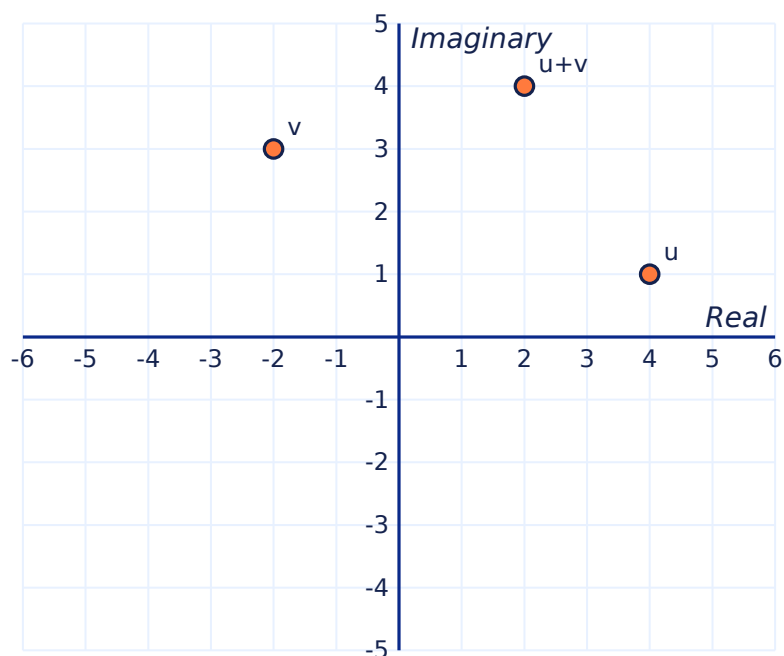
Two complex numbers are equal when their real parts match and their imaginary parts match.

$x + 3 = 9$ gives $x = 6$. $2y - 1 = 7$ gives $y = 4$.

$x = 6, y = 4$

Problem 13 Adding on the complex plane

$u + v = (4 - 2) + (1 + 3)i = 2 + 4i$.



The sum is the fourth vertex of the parallelogram formed with the origin.

$$\mathbf{u + v = 2 + 4i}$$

Problem 14 Squaring a binomial

$$(1 - i)^2 = 1 - 2i + i^2 = 1 - 2i - 1 = -2i.$$

$$\text{Then } (1 - i)^6 = (-2i)^3 = -8i^3 = -8(-i) = 8i.$$

$$\mathbf{(1 - i)^6 = 8i}$$

Problem 15 Checking a solution

$$(3 + 5i)^2 = 9 + 30i - 25 = -16 + 30i.$$

$$-6(3 + 5i) = -18 - 30i.$$

$$\text{Sum: } -16 + 30i - 18 - 30i + 34 = 0.$$

Yes, $3 + 5i$ is a solution.

Problem 16 Sum of four consecutive powers

The remainders on division by 4 are 1, 2, 3, 0, so the values are i , -1 , $-i$, 1.

$$\text{Sum: } i - 1 - i + 1 = \mathbf{0}.$$

Problem 17 Series impedance

Series impedances add: $(12 + 7) + (5 - 9)i$.

$$\mathbf{19 - 4i \text{ ohms}}$$

Problem 18 Voltage in an AC circuit

$$(5 + 2i)(3 - 6i) = 15 - 30i + 6i - 12i^2.$$

$$= 15 - 24i + 12 = 27 - 24i.$$

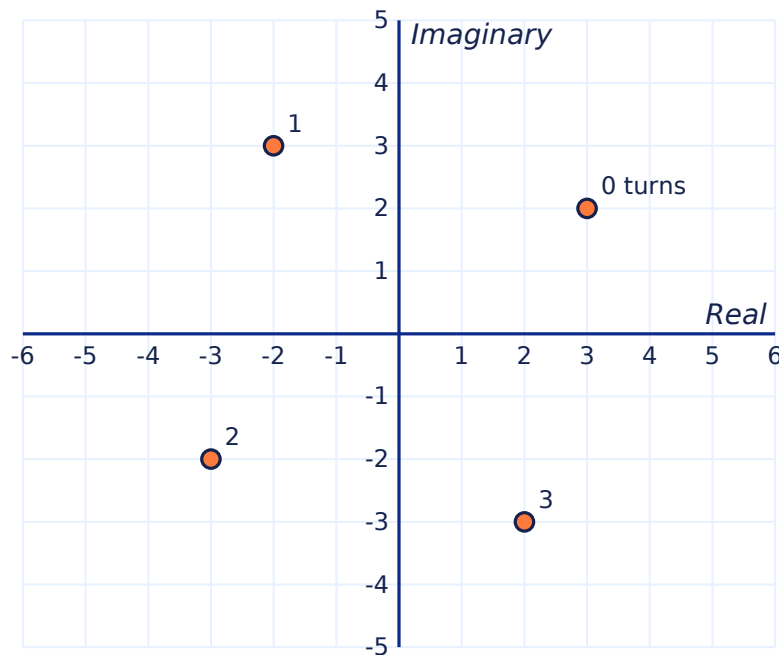
$$\mathbf{27 - 24i \text{ volts}}$$

Problem 19 Quarter turns of a marker

$$(3 + 2i)i = 3i + 2i^2 = -2 + 3i.$$

$$(-2 + 3i)i = -3 - 2i.$$

$$(-3 - 2i)i = 2 - 3i.$$



Each multiplication by i rotates the point a quarter turn; four turns return to the start.

$-2 + 3i$, $-3 - 2i$, $2 - 3i$

Problem 20 A real product

$$\text{Expand: } 6 - 15i + 2ki - 5ki^2 = (6 + 5k) + (2k - 15)i.$$

The imaginary part is 0 when $2k = 15$, so $k = 7.5$. The real part is $6 + 37.5 = 43.5$.

$k = 7.5$ and the product is 43.5