

Quadratics with Complex Solutions

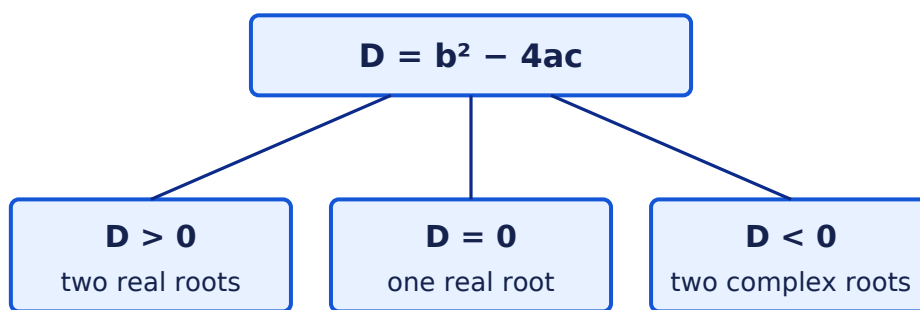
Grade 11 – Algebra 2 math · Lesson · Standards: N-CN.C.7, A-REI.B.4b

Quick answer: The discriminant $D = b^2 - 4ac$ of a quadratic $ax^2 + bx + c = 0$ shows its solutions: $D > 0$ gives two real, $D = 0$ gives one real, $D < 0$ gives two complex conjugates. Apply the quadratic formula and write $\sqrt{D}$ as $i\sqrt{-D}$ to get exact answers.

Standards: N-CN.C.7, A-REI.B.4b · **Level:** Grade 11 Algebra 2

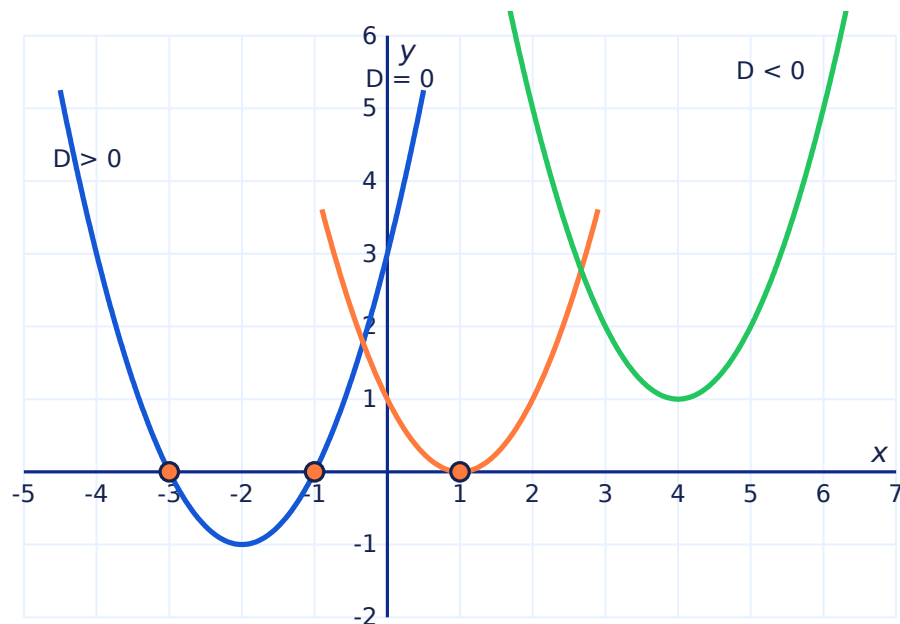
What does the discriminant tell you?

For a quadratic equation $ax^2 + bx + c = 0$ with real coefficients, the **discriminant** is $D = b^2 - 4ac$. Its sign decides what kind of solutions exist before any solving is done.



The sign of D sorts every real quadratic into one of three cases.

Value of D	Solutions	Graph of $y = ax^2 + bx + c$
$D > 0$	Two real numbers	Crosses the x-axis twice
$D = 0$	One repeated real number	Touches the x-axis once
$D < 0$	Two conjugate complex numbers	Never meets the x-axis



Three parabolas, one for each sign of D .

How do you solve a quadratic with complex solutions?

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

When $D < 0$, write $\sqrt{D} = i\sqrt{-D}$ and simplify the radical.

Method

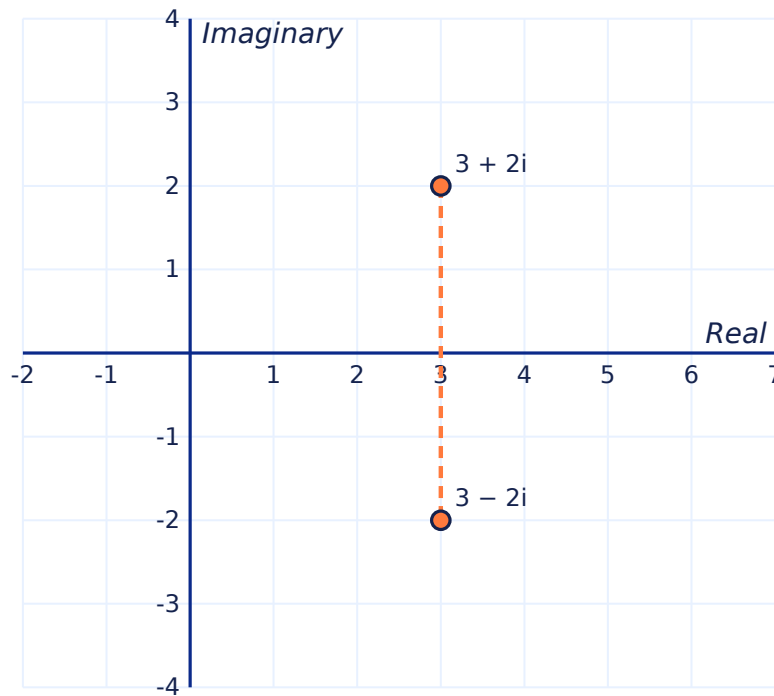
1. Write the equation as $ax^2 + bx + c = 0$ and name a , b , c .
2. Compute $D = b^2 - 4ac$.
3. If $D < 0$, replace $\sqrt{D}$ with $i\sqrt{-D}$ and simplify.
4. Divide every part of the numerator by $2a$ and reduce.

Worked example

Solve $2x^2 - 4x + 5 = 0$. Here $D = 16 - 40 = -24$ and $\sqrt{-24} = 2i\sqrt{6}$. Then $x = \frac{4 \pm 2i\sqrt{6}}{4} = 1 \pm \frac{\sqrt{6}}{2}i$.

Why do complex solutions come in pairs?

The $\pm$ in the formula produces two numbers with the same real part and opposite imaginary parts. Such numbers are **complex conjugates**, and they are mirror images across the real axis.



The solutions $3 + 2i$ and $3 - 2i$ of $x^2 - 6x + 13 = 0$ are conjugates.

Common mistake

Do not stop at $x = \frac{4 \pm \sqrt{(-24)}}{4}$. A root of a negative number must be rewritten with i , and any common factor must be reduced across the whole numerator and denominator.

Mathbot's tip

Check any pair of solutions fast: they add to $-b/a$ and multiply to c/a . For $2x^2 - 4x + 5$, the sum is 2 and the product is $\frac{5}{2}$.

Key points to remember

- $D = b^2 - 4ac$ decides the number and type of solutions.
- If $D < 0$, the parabola never crosses the x -axis and the solutions are non-real.
- Complex solutions of a real quadratic always appear as conjugates $p + qi$ and $p - qi$.
- The quadratic formula gives exact solutions: $x = \frac{-b \pm \sqrt{D}}{2a}$.
- The two solutions sum to $-b/a$ and multiply to c/a .

Quadratics with Complex Solutions: frequently asked questions

What does a negative discriminant mean?

A negative discriminant means the quadratic has no real solutions, so its graph does not cross the x -axis. The equation still has two solutions, and both are non-real complex numbers that are conjugates of each other.

Can a quadratic have just one complex solution?

Not if its coefficients are real. Non-real solutions come in conjugate pairs, so a quadratic with real coefficients has either two non-real solutions or real solutions.

How do you simplify the radical when D is negative?

Factor out -1 first: $\sqrt{-24} = i\sqrt{24}$. Then pull out perfect squares, so $\sqrt{24} = 2\sqrt{6}$ and $\sqrt{-24} = 2i\sqrt{6}$.

Is completing the square or the quadratic formula better for complex roots?

Both always work. Completing the square is fast when the x -coefficient is even, as in $x^2 + 6x + 25 = 0$, while the formula is safer when the coefficients are awkward.

How can you check complex solutions?

Use the sum and product shortcut: the solutions add to $-b/a$ and multiply to c/a . You can also substitute one solution into the equation and confirm that the result is 0.