

Quadratics with Complex Solutions

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: N-CN.C.7, A-REI.B.4b

Each solution shows the discriminant, the simplified radical, and the final answer in bold. Use these steps to find where a sign or a fraction went wrong.

Standards: N-CN.C.7, A-REI.B.4b · **Level:** Grade 11 Algebra 2

Problem 1 Discriminant and type of solutions

- a) $D = (-3)^2 - 4(1)(-10) = 49$: **two distinct real solutions.**
- b) $D = (-8)^2 - 4(1)(16) = 0$: **one repeated real solution.**
- c) $D = (3)^2 - 4(2)(4) = -23$: **two non-real complex solutions (a conjugate pair).**
- d) $D = (-2)^2 - 4(3)(-7) = 88$: **two distinct real solutions.**

Problem 2 Solving with square roots

- a) $x = \pm\sqrt{-49} = \pm 7i$.
- b) $x^2 = -36$, so $x = \pm 6i$.
- c) $3x^2 = -75$, $x^2 = -25$, so $x = \pm 5i$.

Problem 3 Solutions from a squared binomial

- a) $x + 4 = \pm 5i$, so $x = -4 \pm 5i$. **$-4 + 5i$ and $-4 - 5i$**
- b) $(x - 3)^2 = -49$, so $x - 3 = \pm 7i$ and $x = \mathbf{3 \pm 7i}$.

Problem 4 Quadratic formula with a negative discriminant

Here $a = 1$, $b = 10$, $c = 41$.

Discriminant: $D = b^2 - 4ac = (10)^2 - 4(1)(41) = -64$.

$D < 0$, so there are two non-real solutions. $\sqrt{-64} = 8i$.

$$x = \frac{-(-10) \pm \sqrt{(-64)}}{2(1)}$$

$$x = -5 \pm 4i$$

Problem 5 Reading the graph

The vertex (4, 3) is above the x-axis and the parabola opens upward, so the graph never crosses the axis: **0 real solutions**.

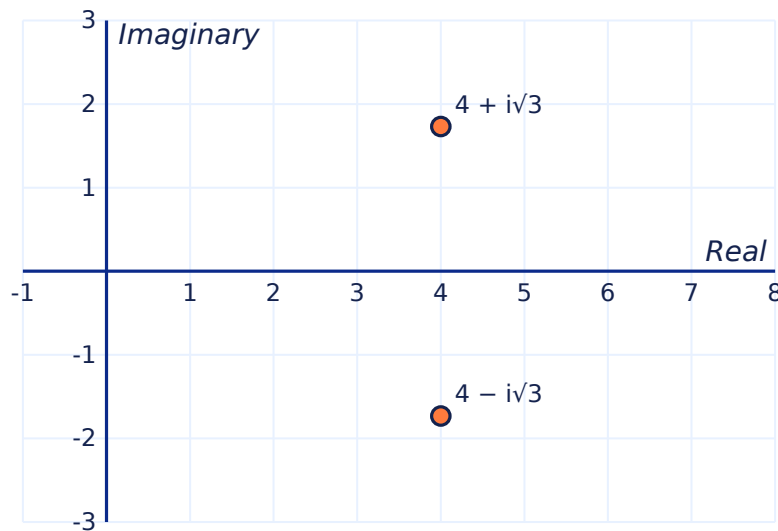
Here $a = 1$, $b = -8$, $c = 19$.

Discriminant: $D = b^2 - 4ac = (-8)^2 - 4(1)(19) = -12$.

$D < 0$, so there are two non-real solutions. $\sqrt{-12} = 2i\sqrt{3}$.

$$x = \frac{-(-8) \pm \sqrt{-12}}{2(1)}$$

$$x = 4 \pm i\sqrt{3}$$



The two non-real solutions.

Problem 6 Completing the square

$x^2 - 12x = -45$. Add 36 to both sides: $(x - 6)^2 = -9$.

$x - 6 = \pm 3i$, so $x = 6 \pm 3i$.

Problem 7 Fractions in the solution

Here $a = 3$, $b = 5$, $c = 4$.

Discriminant: $D = b^2 - 4ac = (5)^2 - 4(3)(4) = -23$.

$D < 0$, so there are two non-real solutions. $\sqrt{-23} = i\sqrt{23}$.

$$x = \frac{(-5) \pm \sqrt{(-23)}}{2(3)}$$

$$x = \frac{(-5 \pm i\sqrt{23})}{6}$$

Problem 8 A parabola that opens downward

The vertex (1, -5) is below the axis and the parabola opens downward, so it never reaches $y = 0$.

Multiply by -1 : $x^2 - 2x + 6 = 0$.

Here $a = 1$, $b = -2$, $c = 6$.

Discriminant: $D = b^2 - 4ac = (-2)^2 - 4(1)(6) = -20$.

$D < 0$, so there are two non-real solutions. $\sqrt{-20} = 2i\sqrt{5}$.

$$x = \frac{-(-2) \pm \sqrt{(-20)}}{2(1)}$$

$$x = 1 \pm i\sqrt{5}$$

Problem 9 A solution with a reduced fraction

Here $a = 5$, $b = -4$, $c = 1$.

Discriminant: $D = b^2 - 4ac = (-4)^2 - 4(5)(1) = -4$.

$D < 0$, so there are two non-real solutions. $\sqrt{-4} = 2i$.

$$x = \frac{-(-4) \pm \sqrt{(-4)}}{2(5)}$$

$$x = \frac{(2 \pm i)}{5}$$

Problem 10 Reducing a radical solution

Here $a = 3$, $b = -6$, $c = 10$.

Discriminant: $D = b^2 - 4ac = (-6)^2 - 4(3)(10) = -84$.

$D < 0$, so there are two non-real solutions. $\sqrt{-84} = 2i\sqrt{21}$.

$$x = \frac{-(-6) \pm \sqrt{(-84)}}{2(3)}$$

$$x = \frac{(3 \pm i\sqrt{21})}{3}$$

Problem 11 Choosing a constant

$$D = (-8)^2 - 4(1)k = 64 - 4k.$$

Non-real solutions need $D < 0$: $64 - 4k < 0$, so $k > 16$.

One repeated real solution needs $D = 0$: $k = 16$.

$k > 16$; $k = 16$

Problem 12 Plotting complex solutions

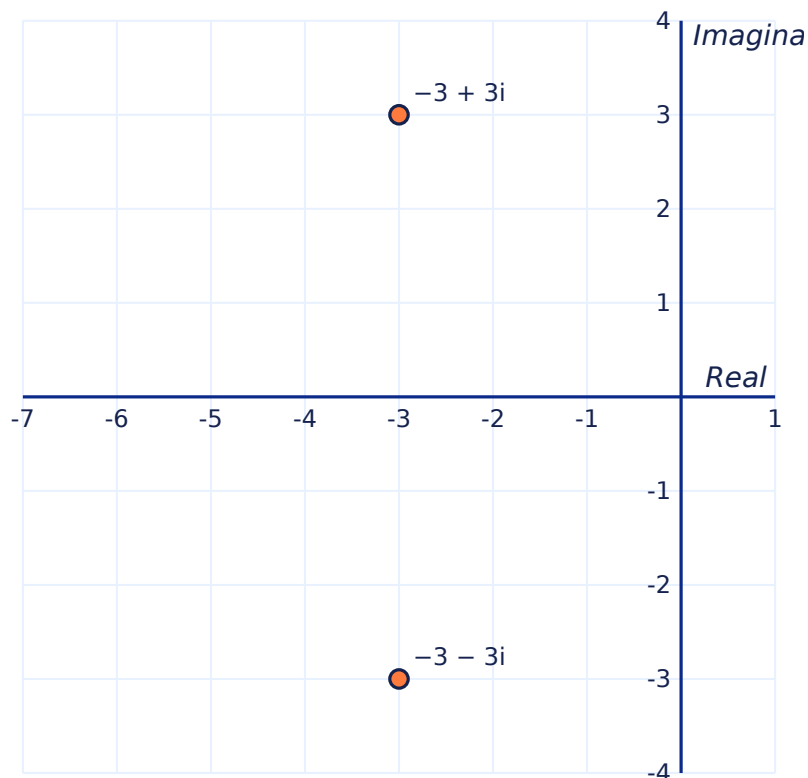
Here $a = 1$, $b = 6$, $c = 18$.

$$\text{Discriminant: } D = b^2 - 4ac = (6)^2 - 4(1)(18) = -36.$$

$D < 0$, so there are two non-real solutions. $\sqrt{-36} = 6i$.

$$x = \frac{(-6) \pm \sqrt{(-36)}}{2(1)}$$

$$\mathbf{x = -3 \pm 3i}$$



The conjugate pair.

Sum: $(-3 + 3i) + (-3 - 3i) = -6 = -(6)/1$. Product: $9 - 9i^2 = 18 = 18/1$.

$-3 + 3i$ and $-3 - 3i$; sum -6 , product 18

Problem 13 Building an equation from solutions

Sum = 14 and product = $49 + 4 = 53$.

$$x^2 - (\text{sum})x + (\text{product}) = 0.$$

$$x^2 - 14x + 53 = 0$$

Problem 14 Which equations have non-real solutions

a) $D = -3$: non-real.

b) $D = 49$: real.

c) $D = 0$: real.

d) $D = -47$: non-real.

Non-real solutions: (a) and (d).

Problem 15 Completing the square with a leading coefficient

a) $x^2 + 6x = -25$, so $(x + 3)^2 = -16$ and $x + 3 = \pm 4i$. **$x = -3 \pm 4i$**

b) Divide by 2: $x^2 - 6x + 13 = 0$. Then $(x - 3)^2 = -4$, $x - 3 = \pm 2i$. **$x = 3 \pm 2i$**

Problem 16 Will the ball reach the height?

Set $-16t^2 + 40t + 5 = 40$, so $16t^2 - 40t + 35 = 0$.

Here $a = 16$, $b = -40$, $c = 35$.

Discriminant: $D = b^2 - 4ac = (-40)^2 - 4(16)(35) = -640$.

$D < 0$, so there are two non-real solutions. $\sqrt{-640} = 8i\sqrt{10}$.

$$t = \frac{-(-40) \pm \sqrt{(-640)}}{2(16)}$$

$$t = \frac{(5 \pm i\sqrt{10})}{4}$$

$D < 0$, so the ball never reaches 40 feet; its maximum is $5 + 25 = 30$ feet.

Problem 17 Profit that cannot happen

Set $-3n^2 + 36n - 90 = 100$, so $3n^2 - 36n + 190 = 0$.

$$D = (-36)^2 - 4(3)(190) = 1296 - 2280 = -984 < 0, \text{ so there is no real } n.$$

The vertex is at $n = 6$: $P(6) = -108 + 216 - 90 = 18$.

Profit never reaches \$100; the maximum is \$18

Problem 18 Sum and product of solutions

Here $a = 2$, $b = -10$, $c = 17$.

$$\text{Discriminant: } D = b^2 - 4ac = (-10)^2 - 4(2)(17) = -36.$$

$D < 0$, so there are two non-real solutions. $\sqrt{-36} = 6i$.

$$x = \frac{-(-10) \pm \sqrt{-36}}{2(2)}$$

$$x = \frac{(5 \pm 3i)}{2}$$

$$\text{Sum: } \frac{(5 + 3i)}{2} + \frac{(5 - 3i)}{2} = 5, \text{ and } -b/a = 10/2 = 5.$$

$$\text{Product: } \frac{(25 - 9i^2)}{4} = \frac{34}{4} = \frac{17}{2} = c/a.$$

Sum 5, product $\frac{17}{2}$; both match

Problem 19 Ranges of a parameter

$$D = k^2 - 64.$$

a) $D < 0$ when $k^2 < 64$, that is $-8 < k < 8$.

b) $D = 0$ when $k = 8$ or $k = -8$.

c) $D > 0$ when $k < -8$ or $k > 8$.

(a) $-8 < k < 8$; (b) $k = \pm 8$; (c) $|k| > 8$

Problem 20 Finding a missing coefficient

Real coefficients force the conjugate $6 + 4i$ to be the other solution.

Sum = 12, so $-b = 12$ and $b = -12$. Product = $36 + 16 = 52$, so $c = 52$.

Other solution $6 + 4i$; $b = -12$, $c = 52$