

Using Logs to Solve Exponent Equations

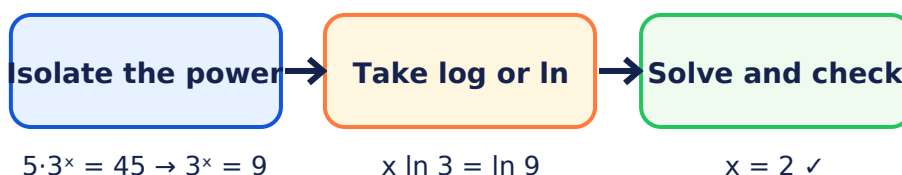
Grade 11 – Algebra 2 math · Lesson · Standards: F-LE.A.4, A-SSE.B.3c

Quick answer: To solve an exponential equation, isolate the power and take a logarithm of both sides so the exponent comes down: if $5 \cdot 3^x = 45$, then $3^x = 9$ and $x = \ln 9 \div \ln 3 = 2$. To solve a logarithmic equation, combine the logs, rewrite in exponential form, and discard answers that make an argument negative.

Standards: F-LE.A.4, A-SSE.B.3c · **Level:** Grade 11 Algebra 2

How do you solve an equation with the unknown in the exponent?

Isolate the power, then apply a logarithm to both sides. The power rule brings the exponent down so that it can be solved like a linear equation.



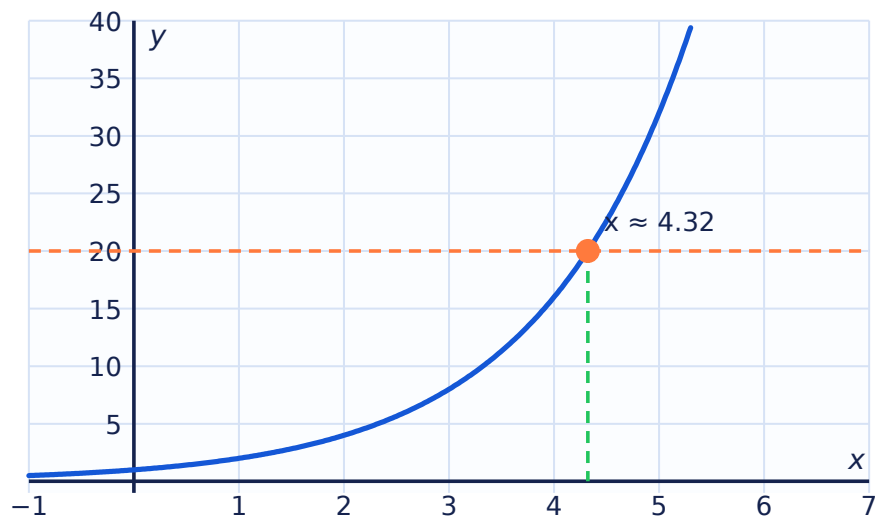
Three steps solve any equation with the unknown in an exponent.

Method: exponential equations

1. Isolate the exponential expression: $a \cdot b^{ct} = d$ becomes $b^{ct} = d/a$.
2. Take log or ln of both sides: $ct \cdot \ln b = \ln(d/a)$.
3. Divide: $t = \ln(d/a) \div (c \ln b)$.
4. Check in the original equation.

Example

Solve $4 \cdot 2^x = 80$. Divide by 4: $2^x = 20$. Then $x \ln 2 = \ln 20$, so $x = \ln 20 \div \ln 2 \approx 2.9957 \div 0.6931 \approx \mathbf{4.32}$.



Solving $2^x = 20$ graphically: the curve meets the line at $x \approx 4.32$.

How do you solve a logarithmic equation?

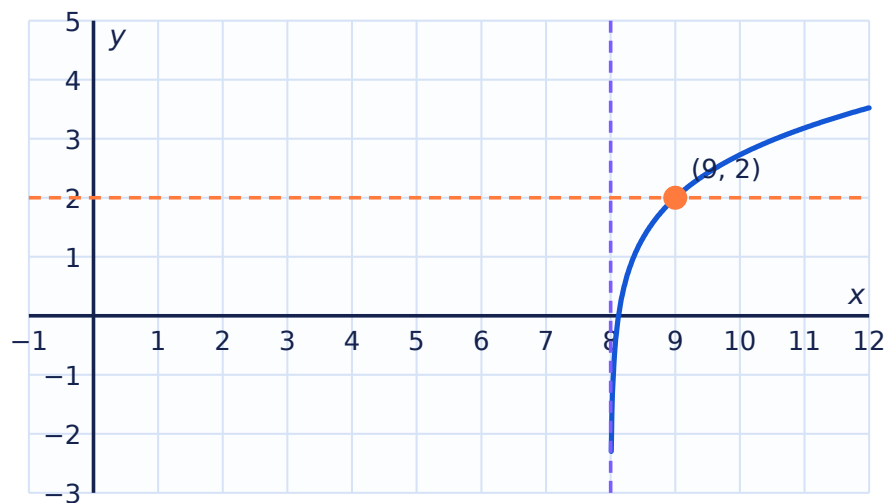
Combine the logs into one, rewrite it in exponential form, and solve. A logarithm is only defined for positive arguments, so every answer must be tested in the original equation.

Two key moves

- $\log_b M = k \iff M = b^k$
- $\log_b M = \log_b N \implies M = N$

Example

Solve $\log_3 x + \log_3(x - 8) = 2$. Combine: $\log_3(x^2 - 8x) = 2$, so $x^2 - 8x = 9$. Then $(x - 9)(x + 1) = 0$. The value $x = -1$ makes the argument negative, so **$x = 9$** .



The sum of logs only exists for $x > 8$, so the root $x = -1$ is extraneous.

Common mistake

Writing $\ln(a + b)$ as $\ln a + \ln b$, or taking the log of only one term. Take the logarithm of an entire side, after isolating the power.

What models use these equations?

Compound interest $A = P(1 + r/n)^{nt}$, continuous growth $A = Pe^{rt}$ and half-life $N = N_0(1/2)^{t/h}$ all have the time t in an exponent. Logarithms solve for t .

Mathbot's tip

Use $\ln$ when the base is e , and either $\log$ or $\ln$ for other bases. Keep full calculator values until the last step, then round once.

Key points to remember

- Isolate the exponential expression before taking logarithms.
- The power rule turns $b^{ct} = d$ into $ct \cdot \ln b = \ln d$.
- $\log_b M = k$ is equivalent to $M = b^k$.
- Always check logarithmic solutions: arguments must be positive.
- Time in growth, decay, interest, pH and decibel problems is found with logarithms.

Using Logs to Solve Exponent Equations: frequently asked questions

When should I use $\ln$ instead of $\log$?

Either works for any exponential equation because the ratio of the two logs is the same. Use $\ln$ when the base is e , and use $\log$ when the base is 10. Mixing them in one fraction is wrong.

What is an extraneous solution in a log equation?

It is a value that satisfies the simplified algebra but makes an argument of a logarithm zero or negative in the original equation. Such a value must be rejected after checking.

Why do I need to isolate the power first?

The logarithm rule applies to a single power. In $4 \cdot 2^x = 80$ the factor 4 sits outside, so taking $\ln$ of both sides without dividing first gives a wrong equation.

How do I find doubling time?

Set the final amount equal to twice the starting amount and solve for time. For continuous growth at rate r the doubling time is $\ln 2 \div r$, and the starting amount cancels.

Can exponential equations be solved without logarithms?

Yes, when both sides can be written with the same base, such as $4^{x+1} = 64$, which gives $x + 1 = 3$. Otherwise logarithms, a graph or a table are required.