

# Using Logs to Solve Exponent Equations

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: F-LE.A.4, A-SSE.B.3c

Every solution shows the isolating step, the logarithm step and the check. Keep extra decimals until the final line, and rewrite any answer that disagrees with the rounding shown here.

**Standards:** F-LE.A.4, A-SSE.B.3c · **Level:** Grade 11 Algebra 2

## Problem 1 Match the bases

$$729 = 3^6, \text{ so } x = 6.$$

## Problem 2 Same base on both sides

$$64 = 4^3, \text{ so } x + 1 = 3 \text{ and } x = 2.$$

## Problem 3 Simplest logarithmic equation

$$\text{Rewrite as } x = 5^3. \quad x = 125.$$

## Problem 4 Negative log value

$$x = 2^{-3} = 1/8. \quad x = 1/8.$$

## Problem 5 Isolate, then match bases

$$\text{Divide by 3: } 2^x = 16 = 2^4. \quad x = 4.$$

## Problem 6 Exponential equation with logs

$$x \ln 7 = \ln 20, \text{ so } x = 2.9957 \div 1.9459 = 1.5395. \quad x \approx 1.54.$$

## Problem 7 Base e equation

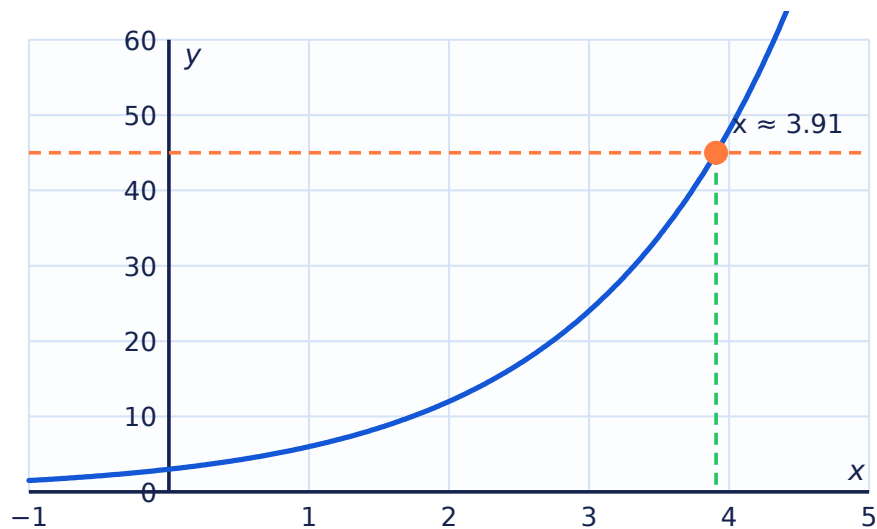
$$\text{Take ln: } 2x = \ln 15 \approx 2.7081, \text{ so } x \approx 1.3540. \quad x \approx 1.35.$$

## Problem 8 Logarithm with a shifted argument

$$\text{Rewrite: } x + 4 = 10^2 = 100. \quad x = 96. \text{ Check: } \log 100 = 2.$$

## Problem 9 Graph of an exponential equation

$$\text{Divide by 3: } 2^x = 15, \text{ so } x = \log_2 15 = \ln 15 \div \ln 2 = 3.9069. \text{ The graph crosses the line just before } x = 4. \quad x = \log_2 15 \approx 3.91.$$



The intersection is at  $x \approx 3.91$ .

### Problem 10 Natural log on a graph

$\ln x = 2$  means  $x = e^2 \approx 7.3891$ .  **$x = e^2 \approx 7.39$ .**

### Problem 11 Condense logs and solve

Combine:  $\ln(x^2 + 2x) = \ln 24$ , so  $x^2 + 2x - 24 = 0$  and  $(x + 6)(x - 4) = 0$ . The value  $-6$  makes  $\ln x$  undefined.  **$x = 4$ .**

### Problem 12 Different bases on each side

Take  $\ln$ :  $(x + 3) \ln 2 = x \ln 5$ . Then  $3 \ln 2 = x(\ln 5 - \ln 2)$ , so  $x = 2.0794 \div 0.9163 = 2.2694$ .  **$x \approx 2.27$ .**

### Problem 13 Logarithm equation leading to a quadratic

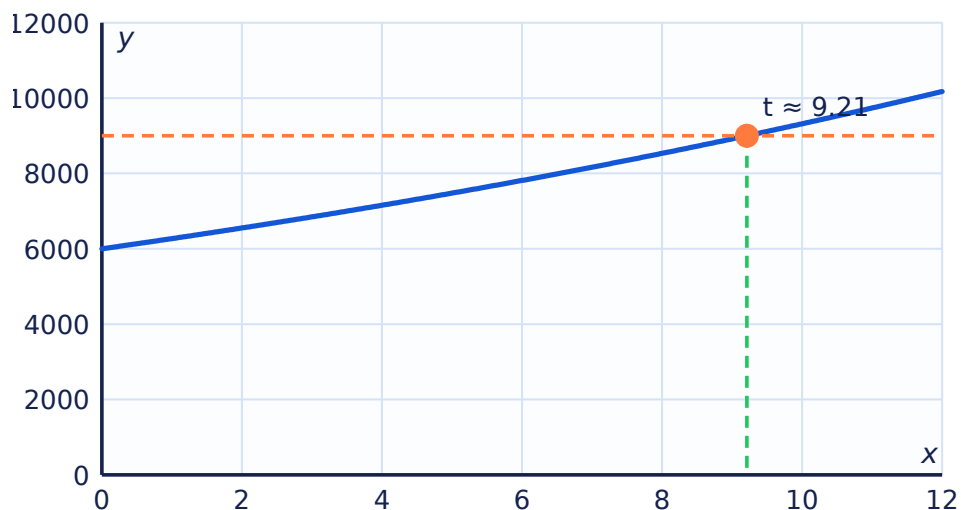
Combine:  $\log(x^2 - 3x) = 1$ , so  $x^2 - 3x = 10$  and  $(x - 5)(x + 2) = 0$ . Since  $x = -2$  is extraneous,  **$x = 5$ .** Check:  $\log 5 + \log 2 = \log 10 = 1$ .

### Problem 14 Equation with a coefficient on the log

Subtract 5:  $3 \log_2(x - 1) = 9$ , so  $\log_2(x - 1) = 3$ . Then  $x - 1 = 8$ .  **$x = 9$ .**

### Problem 15 Savings goal with annual compounding

Solve  $6000(1.045)^t = 9000$ , so  $1.045^t = 1.5$ . Then  $t = \ln 1.5 \div \ln 1.045 = 0.40547 \div 0.04402 = 9.212$ .  **$\approx 9.21$  years.**



The balance reaches \$9,000 at about 9.21 years.

### Problem 16 Medicine half-life

Solve  $80(1/2)^{t/6} = 15$ , so  $(1/2)^{t/6} = 3/16$ . Then  $t/6 = \ln(3/16) \div \ln(1/2) = \log_2(16/3) = 2.4150$ .  $t = 14.490$ .  $\approx$  **14.5 days**.

### Problem 17 Town population growth

Solve  $1200e^{0.035t} = 2000$ , so  $e^{0.035t} = 5/3$ . Then  $0.035t = \ln(5/3) \approx 0.5108$  and  $t \approx 14.60$ . That is during the 15th year after 2020. **2035**.

### Problem 18 Doubling time at 3.2%

Set  $Pe^{0.032t} = 2P$ . Dividing by  $P$  gives  $e^{0.032t} = 2$ , so  $t = \ln 2 \div 0.032 = 21.661$ .  $\approx$  **21.7 years**. The amount  $P$  cancels, so the doubling time is the same for every deposit.

### Problem 19 Hydrogen ion concentration

$-\log[H^+] = 4.3$ , so  $\log[H^+] = -4.3$  and  $[H^+] = 10^{-4.3} \approx 5.0119 \times 10^{-5}$ .  $\approx$   **$5.01 \times 10^{-5}$  moles per liter**.

### Problem 20 Comparing two sound levels

The levels differ by 24 dB. Then  $10 \log(I_1/I_2) = 24$ , so  $\log(I_1/I_2) = 2.4$  and the ratio is  $10^{2.4} = 251.19$ .  $\approx$  **251 times**.