

Exponential Models: Growth, Decay, and Interest

Grade 11 – Algebra 2 math · Lesson · Standards: F-IF.C.8b, F-LE.A.4, F-LE.B.5

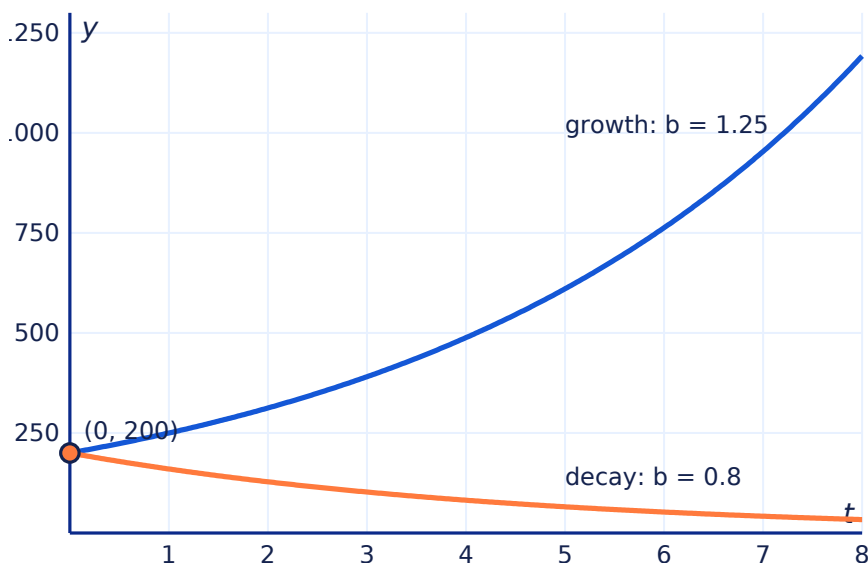
Quick answer: An exponential model $y = a \cdot b^t$ grows if $b > 1$ and decays if $0 < b < 1$. Compound interest uses $A = P(1 + r/n)^{nt}$, continuous growth uses $A = Pe^{rt}$, and half-life problems raise $1/2$ to the power t over the half-life.

Standards: F-IF.C.8b, F-LE.A.4, F-LE.B.5 · **Level:** Grade 11 Algebra 2

What does an exponential model look like?

An **exponential function** has the form $y = a \cdot b^t$, where a is the initial amount and b is the growth or decay factor. If $b > 1$ the quantity grows; if $0 < b < 1$ it decays. A percent change of r per period gives $b = 1 + r$ for growth and $b = 1 - r$ for decay.

Situation	Factor b	Model
Grows 6.5% per year	1.065	$y = a(1.065)^t$
Loses 12% per year	0.88	$y = a(0.88)^t$
Doubles every d periods	$2^{1/d}$	$y = a \cdot 2^{t/d}$
Half-life h	$(1/2)^{1/h}$	$y = a(1/2)^{t/h}$



Both graphs start at 200: growth factor 1.25 rises, decay factor 0.8 falls.

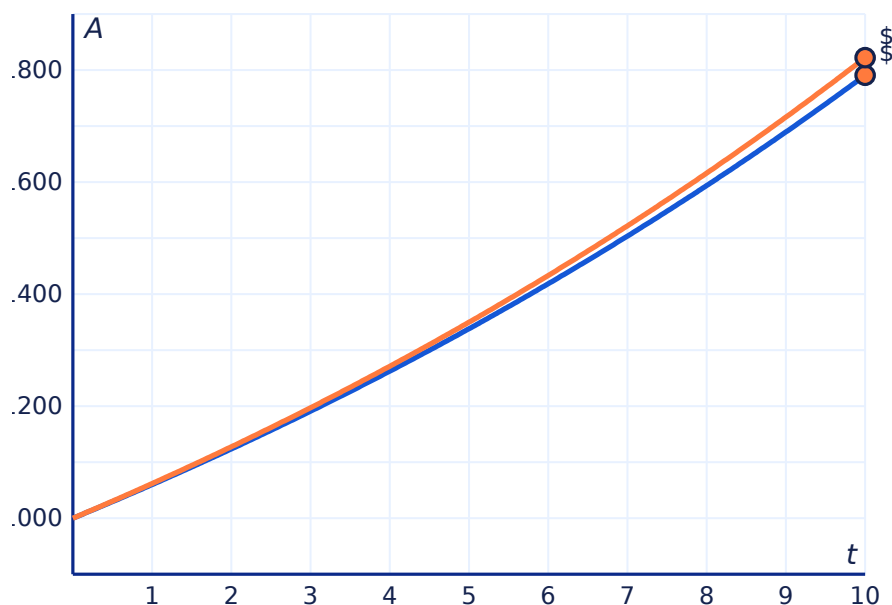
How does compound interest work?

Compound interest formulas

Compounded n times per year: $A = P(1 + \frac{r}{n})^{nt}$.

Compounded continuously: $A = Pe^{rt}$, where $e \approx 2.71828$.

Here P is the principal, r is the annual rate as a decimal, and t is time in years. More frequent compounding earns slightly more, and continuous compounding is the limit.



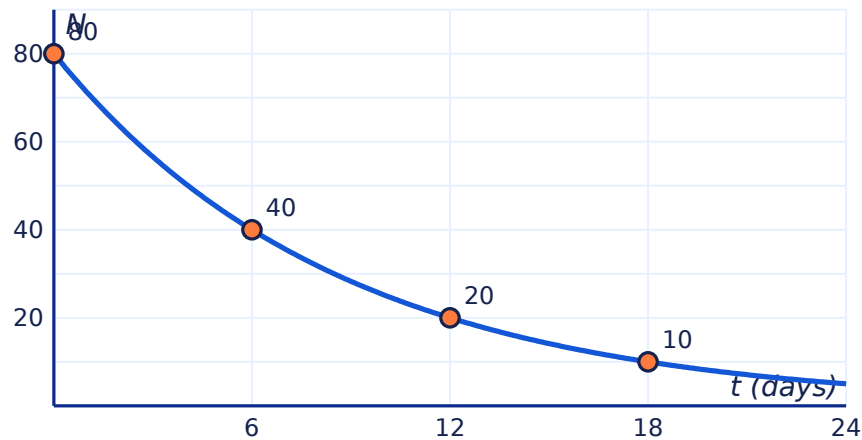
For \$1,000 at 6% for 10 years, annual compounding gives \$1,790.85 and continuous compounding gives \$1,822.12.

Worked example

Find the value of \$4,000 at 3.6% compounded monthly for 5 years. Here $n = 12$ and $nt = 60$: $A = 4,000(1 + 0.036/12)^{60} = 4,000(1.003)^{60} \approx \mathbf{\$4,787.58}$.

What are half-life and doubling time?

A **half-life** is the time for a quantity to fall to half its amount; a **doubling time** is the time for it to double. In both cases the factor 2 or 1/2 is raised to t divided by the period.



With a half-life of 6 days, 80 mg falls to 40, 20, and 10 mg.

Method: build an exponential model

1. Find the initial amount a .
2. Find the factor b from the percent rate, doubling time, or half-life.
3. Write $y = a \cdot b^t$ and check one known value.
4. Substitute to answer the question and attach units.

Common mistake

A 12% decrease uses $b = 0.88$, not 0.12. Also, do not multiply the rate by t : that describes linear change, not exponential change.

Mathbot's tip

Check the factor first: if it is above 1 the graph must rise, and if it is below 1 it must fall.

Key points to remember

- $y = a \cdot b^t$: a is the initial value and b the growth or decay factor.
- A percent increase of r gives $b = 1 + r$; a percent decrease gives $b = 1 - r$.
- Compounded n times a year: $A = P(1 + r/n)^{nt}$. Continuous: $A = Pe^{rt}$.
- Half-life models use $(1/2)^{t/h}$; doubling models use $2^{t/d}$.
- More frequent compounding earns more, but continuous compounding sets an upper limit.

Exponential Models: Growth, Decay, and Interest: frequently asked questions

How do you tell growth from decay in $y = a \cdot b^t$?

Look at b . If b is greater than 1 the quantity grows, and if b is between 0 and 1 it decays. The percent change per period is $|b - 1|$ written as a percent.

What is the number e ?

The number $e \approx 2.71828$ is the base that arises when interest is compounded more and more often. Continuous growth is modeled by $A = Pe^{rt}$, where r is the annual rate as a decimal.

How do you write a half-life model?

Use $N(t) = N_0(1/2)^{t/h}$, where N_0 is the starting amount and h is the half-life. After each period of length h the amount is cut in half.

Why does compounding more often earn more?

Interest earned in each period is added to the balance and then earns interest itself. With more compounding periods this happens sooner, so the balance is slightly higher, but the gain shrinks toward the continuous limit.

What do the parameters of an exponential model mean?

The parameter a is the value at $t = 0$. The factor b tells how much the value is multiplied by during one period, so $b - 1$ is the percent rate of change per period.