

Exponential Models: Growth, Decay, and Interest

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: F-IF.C.8b, F-LE.A.4, F-LE.B.5

Each solution names the initial amount and the factor, shows the substitution, and states the result with units. Money amounts are rounded to the nearest cent.

Standards: F-IF.C.8b, F-LE.A.4, F-LE.B.5 · **Level:** Grade 11 Algebra 2

Problem 1 Growth or decay

- a) Factor $1.04 > 1$: **growth of 4%.**
- b) Factor $0.85 < 1$: **decay of 15%.**
- c) Factor 2: **growth of 100%.**
- d) Factor 0.97: **decay of 3%.**

Problem 2 A growing town

The growth factor is $1 + 0.03 = 1.03$, so $P(t) = 8,400(1.03)^t$.

$$P(5) = 8,400(1.03)^5 \approx 9,738.$$

$P(t) = 8,400(1.03)^t$; about 9,738 people

Problem 3 A depreciating car

The decay factor is $1 - 0.11 = 0.89$, so $V(t) = 31,500(0.89)^t$.

$$V(3) = 31,500(0.89)^3 \approx \$22,206.52.$$

$V(t) = 31,500(0.89)^t$; about \$22,206.52

Problem 4 Reading an exponential graph

Initial value $a = 200$ and growth factor $b = 1.25$, which is 25% growth per period.

$$y(2) = 200(1.5625) = 312.50. \quad y(3) = 200(1.953125) = 390.63.$$

Initial value 200; factor 1.25; $y(2) = 312.50$; $y(3) = 390.63$

Problem 5 Evaluating powers of e

a) $e^{0.5} \approx 1.649$.

b) $e^{-1} \approx 0.368$.

c) $e^{2.3} \approx 9.974$.

1.649; 0.368; 9.974

Problem 6 Annual compounding

$$A = 2,500(1 + 0.045)^6 = 2,500(1.045)^6 \approx \$3,255.65.$$

\$3,255.65

Problem 7 Monthly compounding

$$n = 12 \text{ and } nt = 60: A = 4,000(1 + 0.036/12)^{60} = 4,000(1.003)^{60} \approx \$4,787.58.$$

\$4,787.58

Problem 8 Continuous compounding

$$A = 1,500e^{0.05 \cdot 8} = 1,500e^{0.4} \approx \$2,237.74.$$

\$2,237.74

Problem 9 Comparing compounding frequencies

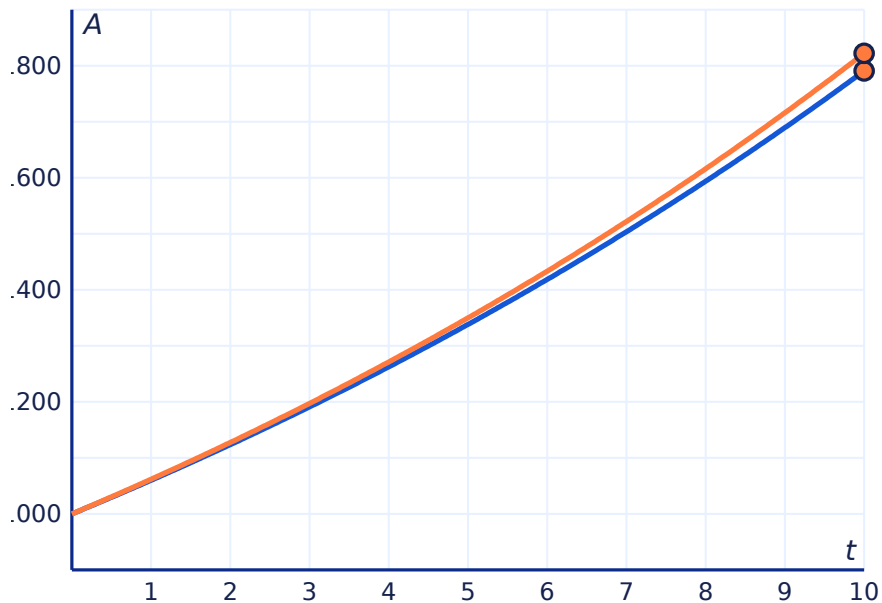
Annually: $1,000(1.06)^{10} \approx \$1,790.85$.

Quarterly: $1,000(1.015)^{40} \approx \$1,814.02$.

Monthly: $1,000(1.005)^{120} \approx \$1,819.40$.

Continuously: $1,000e^{0.6} \approx \$1,822.12$.

More frequent compounding gives more, but the differences are small.



The top two values at 10 years.

\$1,790.85; \$1,814.02; \$1,819.40; \$1,822.12

Problem 10 Interpreting parameters

15,000 is the initial value of the van, in dollars. The factor 0.9 means the value is multiplied by 0.9 each year, a loss of 10% per year.

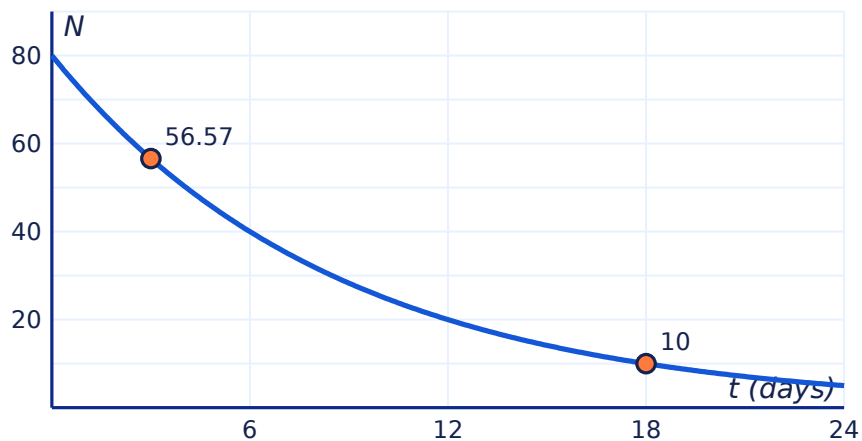
$$V(2) = 15,000(0.81) = \$12,150.00.$$

Initial value \$15,000; loses 10% yearly; \$12,150.00

Problem 11 Half-life

After 18 days there are 3 half-lives: $80 \div 2 \div 2 \div 2 = 10$. Check: $80(1/2)^3 = 10$.

After 3 days: $80(1/2)^{0.5} = 80/\sqrt{2} \approx 56.57$.



Amounts after 3 and 18 days.

10 mg after 18 days; about 56.57 mg after 3 days

Problem 12 Doubling time

$$N(t) = 500 \cdot 2^{t/3}.$$

$$\text{After 12 hours: } 500 \cdot 2^4 = 8,000.$$

$$\text{After 5 hours: } 500 \cdot 2^{5/3} \approx 1,587.$$

$N(t) = 500 \cdot 2^{t/3}$; 8,000 cells; about 1,587 cells

Problem 13 Changing the time unit

$$(1.5)^{t/2} = (1.5^{1/2})^t, \text{ so } b = \sqrt{1.5} \approx 1.2247.$$

The model is $y = 200(1.2247)^t$, an increase of 22.5% per year.

$y \approx 200(1.2247)^t$; about 22.5% per year

Problem 14 Model from a table

Ratios: $460/400 = 1.15$, $529/460 = 1.15$, $608.35/529 = 1.15$. A constant ratio means exponential growth.

15% growth per year; $V(t) = 400(1.15)^t$

Problem 15 Doubling an investment

$$A(t) = 5,000(1.06)^t.$$

$$t = 10: \$8,954.24.$$

$$t = 11: \$9,491.49.$$

$$t = 12: \$10,060.98.$$

The balance first reaches \$10,000 during year 12.

12 years

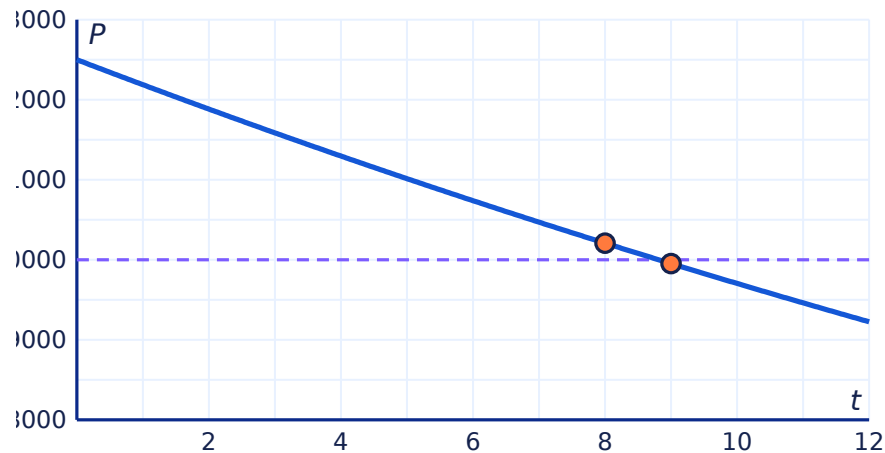
Problem 16 A shrinking town

$$P(t) = 12,500(0.975)^t.$$

$$t = 7: 10,470.$$

$$t = 8: 10,208.$$

$$t = 9: 9,953.$$



The level 10,000 is crossed between years 8 and 9.

$$P(t) = 12,500(0.975)^t; \text{ below } 10,000 \text{ in year } 9$$

Problem 17 Carbon dating

$$11,460 = 2 \times 5,730, \text{ so there are 2 half-lives: } (1/2)^2 = 1/4.$$

$$2,865 \text{ is half of a half-life: } (1/2)^{0.5} \approx 0.7071, \text{ about } 70.7\%.$$

One quarter; about 70.7%

Problem 18 Effective annual yield

$$\text{Monthly: } (1.005)^{12} - 1 \approx 0.06168, \text{ so the yield is about } 6.168\%.$$

$$\text{Continuous: } e^{0.06} - 1 \approx 0.06184, \text{ about } 6.184\%.$$

Monthly 6.168%; continuous 6.184% (slightly higher)

Problem 19 Choosing between two offers

$$A: 3,000(1 + 0.05/4)^{28} = 3,000(1.0125)^{28} \approx \$4,247.98.$$

$$B: 2,950e^{0.052 \cdot 7} = 2,950e^{0.364} \approx \$4,245.27.$$

Offer A is greater: \$4,247.98 compared with \$4,245.27

Problem 20 Model from two data points

Divide the two equations: $\frac{ab^3}{ab} = b^2 = 399.3/330 = 1.21$, so $b = 1.1$.

Then $a(1.1) = 330$ gives $a = 300$.

$$y(5) = 300(1.1)^5 = 483.153.$$

$a = 300$, $b = 1.1$; $y(5) \approx 483.15$