

Transformations of Functions and Their Graphs

Grade 11 – Algebra 2 math · Lesson · Standards: F-BF.B.3, F-IF.C.7b

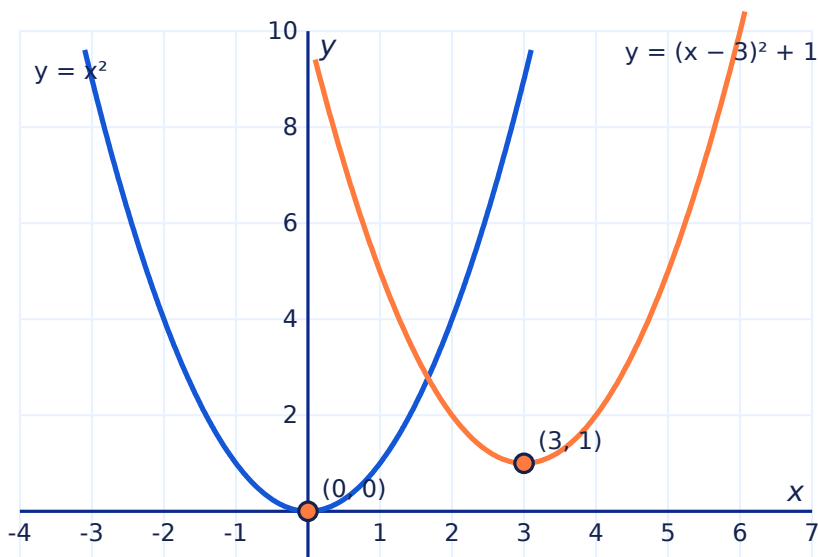
Quick answer: Transformations change a parent graph predictably: $f(x) + k$ shifts up, $f(x - h)$ shifts right, $-f(x)$ reflects across the x-axis, $af(x)$ stretches vertically, and $f(bx)$ compresses horizontally. Even functions are symmetric about the y-axis; odd functions about the origin.

Standards: F-BF.B.3, F-IF.C.7b · **Level:** Grade 11 Algebra 2

How do shifts and reflections change a graph?

Starting from a parent function $y = f(x)$, each change to the equation moves or flips the graph in a predictable way. Changes inside the parentheses act on x and work "backwards"; changes outside act on y .

Equation	Effect on the graph of f
$y = f(x) + k$	Shift up k (down if $k < 0$)
$y = f(x - h)$	Shift right h (left if $h < 0$)
$y = -f(x)$	Reflect across the x-axis
$y = f(-x)$	Reflect across the y-axis
$y = af(x)$	Stretch vertically by a if $ a > 1$; shrink if $0 < a < 1$
$y = f(bx)$	Compress horizontally by $1/b$ if $ b > 1$; stretch if $0 < b < 1$



$y = (x - 3)^2 + 1$ is $y = x^2$ shifted right 3 and up 1.

What are the key parent functions?

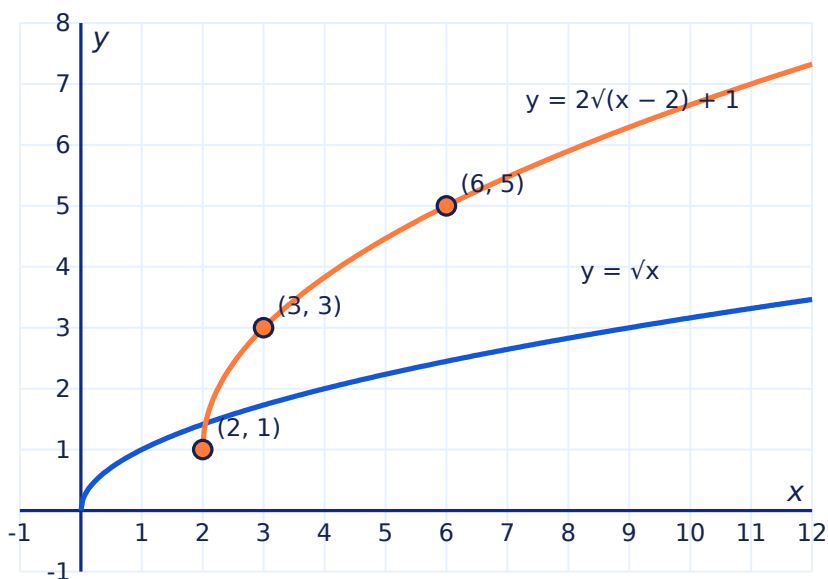
Parent	Rule	Domain	Range
Quadratic	$f(x) = x^2$	All reals	$y \geq 0$
Absolute value	$f(x) = x $	All reals	$y \geq 0$
Square root	$f(x) = \sqrt{x}$	$x \geq 0$	$y \geq 0$
Cube root	$f(x) = \sqrt[3]{x}$	All reals	All reals
Exponential	$f(x) = 2^x$	All reals	$y > 0$

Method: describe a transformed graph

1. Identify the parent function.
2. Locate the key point (vertex, start point, center, or asymptote) of the parent.
3. Apply stretches and reflections, then the shifts.
4. Move the key point and plot a few more points.

Worked example

Graph $g(x) = 2\sqrt{x - 2} + 1$. The parent $\sqrt{x}$ starts at $(0, 0)$. The shift moves the start to $(2, 1)$. The stretch by 2 turns the point $(1, 1)$ of the parent into $(3, 3)$ and $(4, 2)$ into $(6, 5)$.

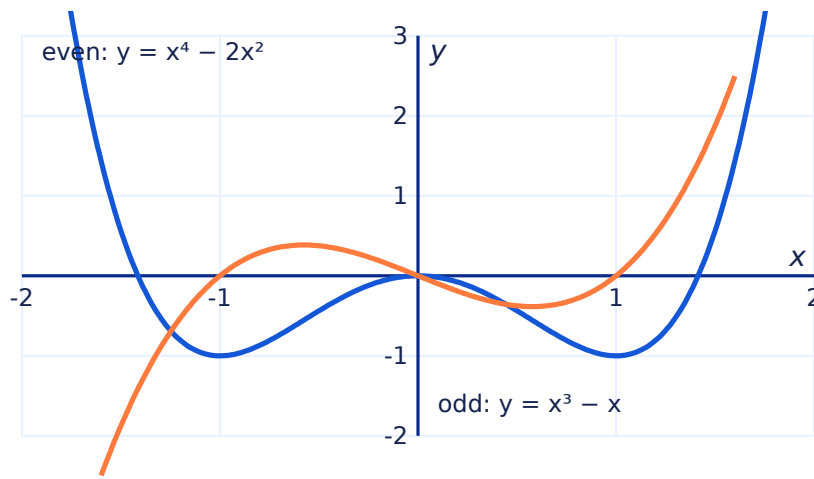


$g(x) = 2\sqrt{x - 2} + 1$ starts at $(2, 1)$ and rises twice as steeply as $\sqrt{x}$.

What are even and odd functions?

Even and odd

A function is **even** if $f(-x) = f(x)$ for every x ; its graph is symmetric about the y -axis. It is **odd** if $f(-x) = -f(x)$; its graph is symmetric about the origin.



$x^4 - 2x^2$ is even and $x^3 - x$ is odd.

Common mistake

Horizontal changes work opposite to what the sign suggests: $f(x + 3)$ moves the graph left 3, not right. Also, a vertical stretch multiplies the y -values, so the key point moves up or down, not sideways.

Mathbot's tip

Track one key point: apply the horizontal changes to its x -coordinate and the vertical changes to its y -coordinate.

Key points to remember

- Outside changes act on y : $f(x) + k$, $af(x)$, and $-f(x)$.
- Inside changes act on x and work backwards: $f(x - h)$ shifts right h .
- A key point of the parent, such as a vertex or start point, shows the move in one step.
- Square root and cube root graphs start at or are centered on the shifted key point.
- Even: $f(-x) = f(x)$. Odd: $f(-x) = -f(x)$.

Transformations of Functions and Their Graphs: frequently asked questions

Why does $f(x - 3)$ shift the graph to the right?

The output of $f(x - 3)$ at $x = 5$ equals $f(2)$, which the parent produces at $x = 2$. So every point of the parent moves 3 units right to produce the new graph.

What is the difference between $f(-x)$ and $-f(x)$?

$-f(x)$ changes the sign of the output, reflecting the graph across the x -axis. $f(-x)$ changes the sign of the input, reflecting the graph across the y -axis.

How do you find the domain of a transformed square root function?

Set the expression under the root greater than or equal to 0. For $y = \sqrt{x + 7} - 2$, $x + 7 \geq 0$ gives $x \geq -7$.

How can you tell if a function is even or odd from its equation?

Replace x with $-x$ and simplify. If you get the original function it is even; if you get its opposite it is odd; otherwise it is neither.

Does the order of transformations matter?

Yes, a stretch or reflection should be applied before a vertical shift. For $y = 2f(x) + 3$, double the outputs first and then add 3, which gives a different graph from adding 3 first and then doubling.