

Transformations of Functions and Their Graphs

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: F-BF.B.3, F-IF.C.7b

Each solution names the parent function, applies the transformations in order, and tracks a key point. Compare the points on each graph with the moved key point.

Standards: F-BF.B.3, F-IF.C.7b · **Level:** Grade 11 Algebra 2

Problem 1 Describing shifts

- a) **Up 5.**
- b) **Right 4** (the shift is opposite the sign inside).
- c) **Left 6.**
- d) **Down 2.**

Problem 2 Writing an equation for a shift

Left 3 replaces x with $x + 3$. Down 6 subtracts 6.

$$y = |x + 3| - 6$$

Problem 3 Evaluating transformed functions

$$3f(16) = 3 \cdot 4 = 12; 3f(25) = 3 \cdot 5 = 15; 3f(36) = 3 \cdot 6 = 18.$$

12, 15, 18

Problem 4 Equation from a graph

The vertex moved from $(0, 0)$ to $(2, 1)$: right 2 and up 1.

$$g(x) = |x - 2| + 1$$

Problem 5 Even, odd, or neither

a) $f(-x) = x^6 - 4x^2 = f(x)$: **even.**

b) $g(-x) = -x^5 + 9x = -g(x)$: **odd**.

c) $h(-x) = x^2 + 3x$, which is neither $h(x)$ nor $-h(x)$: **neither**.

Problem 6 Reflections

$g(x) = -\sqrt{x}$ has the point $(4, -2)$: reflection across the x-axis.

$h(x) = \sqrt{-x}$ has the point $(-4, 2)$: reflection across the y-axis.

$(4, -2)$ and $(-4, 2)$

Problem 7 Stretches and shrinks

The factor $\frac{1}{2}$ shrinks vertically by half, the minus sign reflects across the x-axis, and $+3$ shifts up 3.

$$g(4) = -\frac{1}{2}(16) + 3 = -8 + 3 = -5.$$

Reflect, shrink by $\frac{1}{2}$, up 3; $g(4) = -5$

Problem 8 Several transformations at once

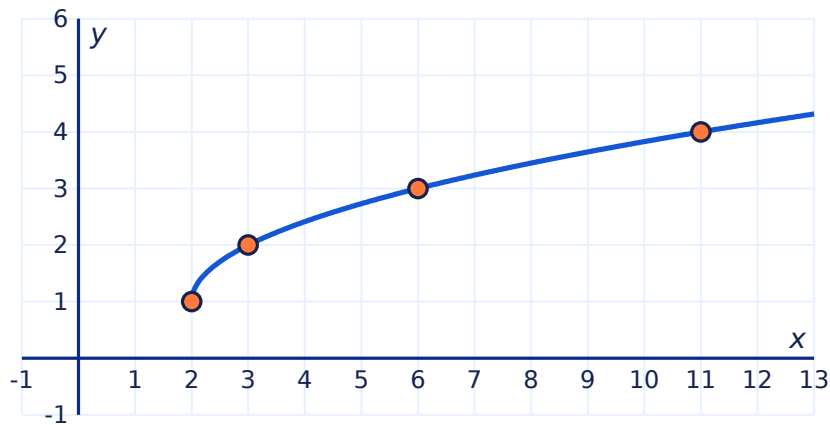
Reflection across the x-axis, vertical stretch by 2, left 1, up 4.

The vertex $(0, 0)$ moves to $(-1, 4)$.

Vertex $(-1, 4)$

Problem 9 Graphing a square root function

The parent $\sqrt{x}$ starts at $(0, 0)$, so the graph starts at $(2, 1)$. Points: $x = 3$ gives 2, $x = 6$ gives 3, $x = 11$ gives 4.

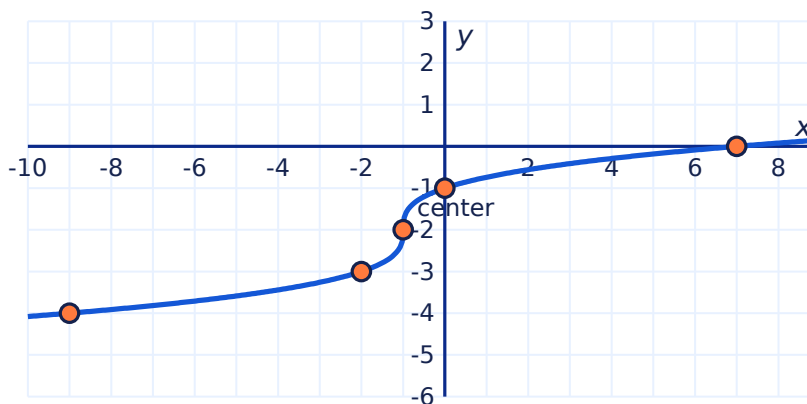


The completed graph.

Start (2, 1); domain $x \geq 2$; range $y \geq 1$

Problem 10 Graphing a cube root function

The parent passes through (0, 0), (1, 1), (-1, -1), (8, 2), (-8, -2). Shift left 1 and down 2: (-1, -2), (0, -1), (-2, -3), (7, 0), (-9, -4).



The completed graph.

Domain: all real numbers; range: all real numbers

Problem 11 An exponential transformation

Right 1 and up 4. The asymptote $y = 0$ moves up to $y = 4$.

$$g(0) = 5^{-1} + 4 = \frac{1}{5} + 4 = \frac{21}{5}.$$

Right 1, up 4; $y = 4$; (0, $\frac{21}{5}$)

Problem 12 Domain and range

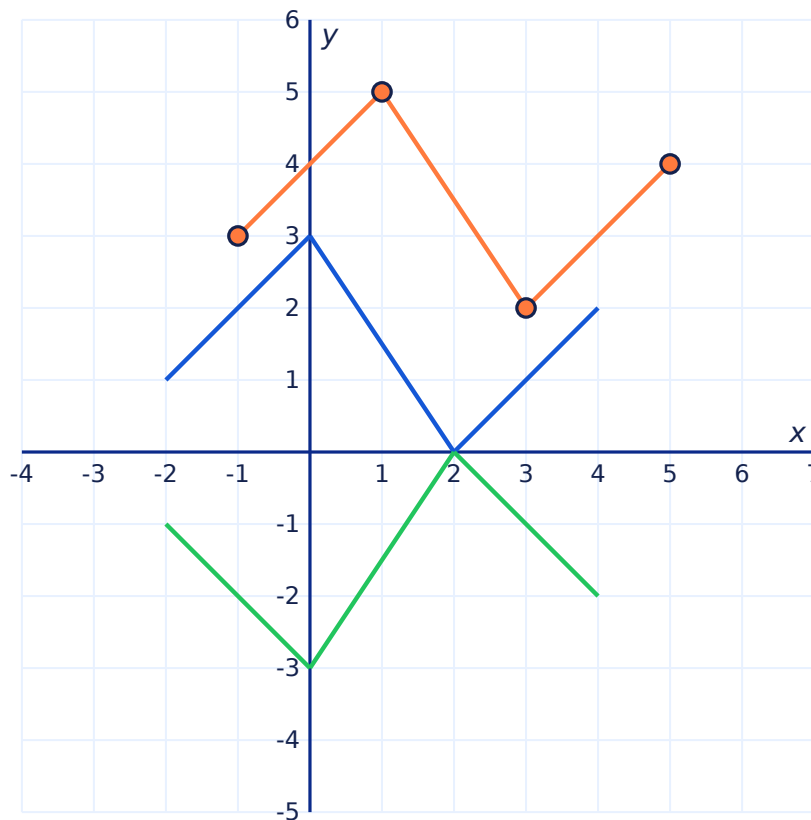
Domain: $x - 4 \geq 0$, so $x \geq 4$. The root is at least 0, so $-\sqrt{x - 4} \leq 0$ and $g(x) \leq 5$.

Domain $x \geq 4$; range $y \leq 5$

Problem 13 Transforming a polygonal graph

g shifts every point right 1 and up 2: $(-1, 3)$, $(1, 5)$, $(3, 2)$, $(5, 4)$.

h changes the sign of every y -coordinate: $(-2, -1)$, $(0, -3)$, $(2, 0)$, $(4, -2)$.



f (blue), g (orange), h (green).

g : $(-1, 3)$, $(1, 5)$, $(3, 2)$, $(5, 4)$; h : $(-2, -1)$, $(0, -3)$, $(2, 0)$, $(4, -2)$

Problem 14 Equation of a parabola from its graph

$h = -2$ and $k = -3$, so $g(x) = a(x + 2)^2 - 3$. At $(0, 1)$: $1 = 4a - 3$, so $a = 1$.

$g(x) = (x + 2)^2 - 3$

Problem 15 Proving even and odd

a) $f(-x) = \frac{(-x)^3}{((-x)^2 + 4)} = \frac{-x^3}{(x^2 + 4)} = -f(x)$: **odd**.

b) $g(-x) = |-x| + (-x)^4 = |x| + x^4 = g(x)$: **even**.

c) $h(-x) = x^4 - x^3$. This is not $h(x) = x^4 + x^3$, and not $-h(x)$: **neither**.

Problem 16 Three launches of a ball

Second ball: shift up 20; the vertex moves from (2, 64) to (2, 84), so the maximum is 84 feet.

Third ball: shift right 0.5; the vertex moves to (2.5, 64), so the maximum is still 64 feet, reached 0.5 second later.



The three maximums.

First 64 ft at 2 s; second 84 ft at 2 s; third 64 ft at 2.5 s

Problem 17 Delayed video views

A 3-day delay is a shift right 3: $V_2(t) = 500 \cdot 2^{t-3}$.

On day 5: $500 \cdot 2^2 = 500 \cdot 4 = 2,000$.

With 800 extra views, add 800: $V_3(t) = 500 \cdot 2^{t-3} + 800$.

$V_2(t) = 500 \cdot 2^{t-3}$; 2,000 views; $V_3(t) = 500 \cdot 2^{t-3} + 800$

Problem 18 Weekly donations

Vertical stretch by 2, right 1, up 3.

$$D(10) = 2\sqrt{9} + 3 = 9. D(5) = 2\sqrt{4} + 3 = 7.$$

$$2\sqrt{(w - 1) + 3} = 11 \text{ gives } \sqrt{(w - 1) + 3} = 5.5, \text{ so } (w - 1) + 3 = 30.25 \text{ and } w = 28.25.$$

$$\mathbf{D(10) = 9; D(5) = 7; week 17}$$

Problem 19 Writing and using a transformed function

$$g(x) = -3|x - 2| - 1.$$

$$g(-1) = -3(3) - 1 = -10. g(2) = -3(0) - 1 = -1. g(5) = -3(3) - 1 = -10.$$

The maximum value is the vertex height -1 , which is below 0 , so the graph never reaches the x -axis.

$$\mathbf{g(x) = -3|x - 2| - 1; -10, -1, -10; no x-intercepts}$$

Problem 20 A horizontal change that is really vertical

$$g(x) = \sqrt{4x - 12} = \sqrt{4(x - 3)} = 2\sqrt{x - 3}.$$

This is f shifted right 3 and stretched vertically by 2 . The domain needs $4x - 12 \geq 0$, so $x \geq 3$.

$$\mathbf{g(x) = 2\sqrt{(x - 3)}; \text{ domain } x \geq 3}$$