

Graphing Waves: Sine and Cosine Models

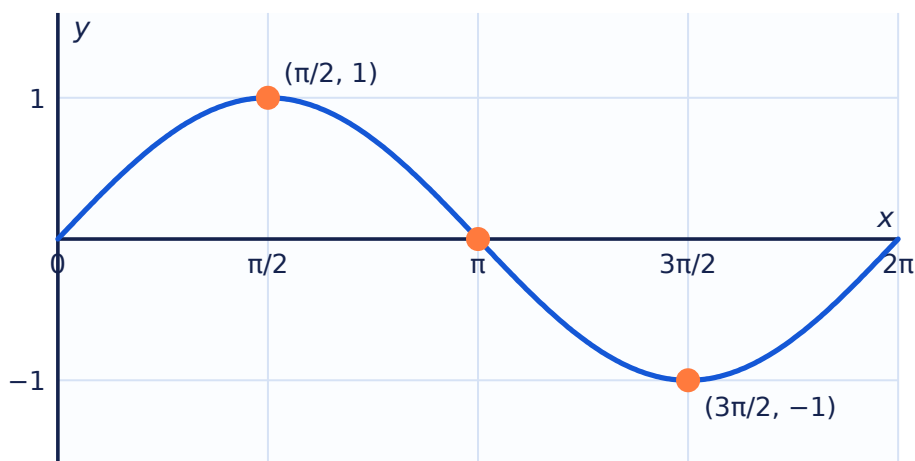
Grade 11 – Algebra 2 math · Lesson · Standards: F-TF.B.5, F-IF.C.7e

Quick answer: The graph of $y = A \sin(B(x - C)) + D$ is a wave with amplitude $|A|$, period $2\pi/|B|$, phase shift C and midline $y = D$. Cosine has the same shape, shifted left by $\pi/2$. To model a repeating situation, set the midline to the average of the highest and lowest values and the amplitude to half their difference.

Standards: F-TF.B.5, F-IF.C.7e · **Level:** Grade 11 Algebra 2

What are the features of a sine or cosine graph?

The graphs of sine and cosine repeat the same wave forever. One full wave is a period, and the basic period is 2π .



One period of $y = \sin x$: zeros at $0, \pi, 2\pi$; maximum at $\pi/2$; minimum at $3\pi/2$.

General form

$$y = A \sin(B(x - C)) + D$$

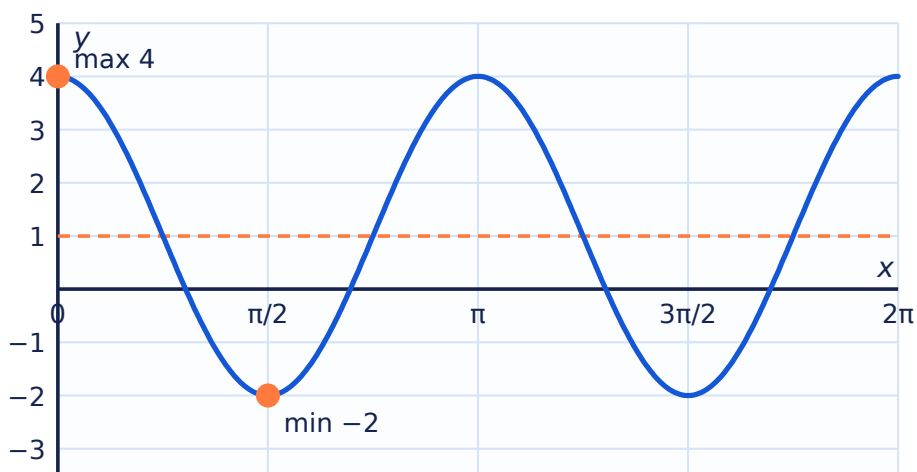
- Amplitude = $|A|$, the distance from the midline to a peak
- Period = $2\pi/|B|$
- Phase shift = C (right if positive)
- Midline: $y = D$
- Maximum = $D + |A|$, minimum = $D - |A|$

The same rules hold for cosine. A negative A flips the wave upside down.

How do you read an equation from a graph?

Method

1. Midline: average of the maximum and minimum.
2. Amplitude: half the difference between maximum and minimum.
3. Period: distance between two peaks, then $B = 2\pi \div \text{period}$.
4. Choose cosine if the graph starts at a maximum on the y-axis, or sine if it starts on the midline going up.



$y = 3 \cos 2x + 1$: amplitude 3, midline $y = 1$, period π .

Example

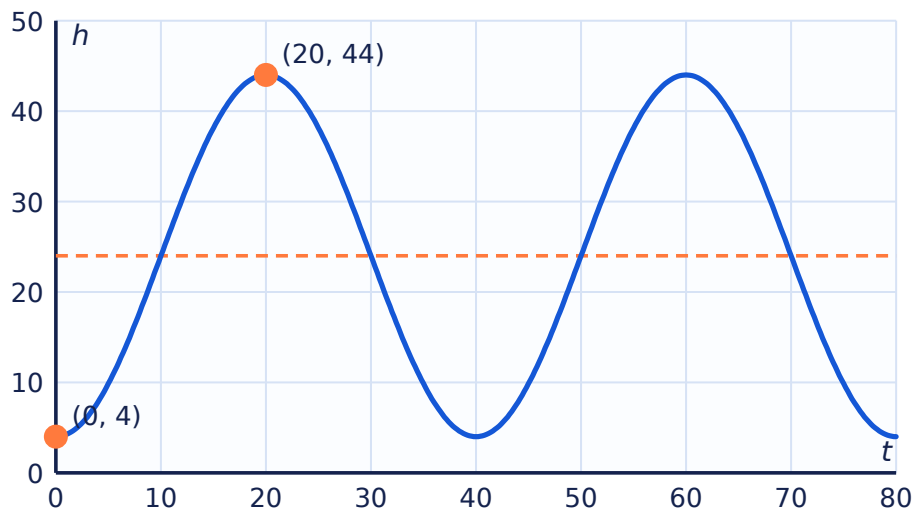
The graph above has maximum 4 and minimum -2 . Midline $= (4 + (-2))/2 = 1$. Amplitude $= (4 - (-2))/2 = 3$. Peaks at 0 and π give period π , so $B = 2\pi/\pi = 2$. **Equation: $y = 3 \cos 2x + 1$.**

Common mistake

Using B as the period. The period is $2\pi/B$, so doubling B halves the period.

How are trigonometric functions used to model real situations?

Any repeating process can be modeled: tides, daylight, temperature and a Ferris wheel. Match the maximum and minimum to the midline and amplitude, then use the time of one cycle to find B .



Height of a rider (feet) versus time (seconds): midline 24, amplitude 20, period 40.

Mathbot's tip

Sketch the midline first and mark the highest and lowest points. The equation almost writes itself.

Key points to remember

- Amplitude is $|A|$, and the maximum and minimum are $D \pm |A|$.
- Period $= 2\pi/|B|$, so a larger B gives a shorter wave.
- Phase shift C moves the graph right when C is positive.
- Sine starts on the midline; cosine starts at its maximum.
- Tides, daylight hours and Ferris wheels are modeled by shifted sine or cosine functions.

Graphing Waves: Sine and Cosine Models: frequently asked questions

How do I find the period of $y = \sin(3x)$?

Divide 2π by the absolute value of B . Here $B = 3$, so the period is $2\pi/3$. The graph completes three full waves between 0 and 2π .

What is the difference between amplitude and midline?

The midline is the horizontal line halfway between the maximum and minimum. The amplitude is the vertical distance from the midline to the maximum, or half the total height of the wave.

How can I tell if a graph is sine or cosine?

Cosine starts at its maximum when $x = 0$, while sine starts on the midline. Because $\cos x = \sin(x + \pi/2)$, either function can describe the same wave with the right phase shift.

What does a negative A do to the graph?

It reflects the wave across the midline. The amplitude is still $|A|$ because it is a distance, which cannot be negative. For example, $y = -2 \cos x$ starts at its minimum.

How do I model a Ferris wheel with a cosine function?

Use the center height for D and the radius for A . The period is the time for one rotation. A rider who starts at the bottom follows $y = -A \cos(2\pi t/\text{period}) + D$.