

Graphing Waves: Sine and Cosine Models

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: F-TF.B.5, F-IF.C.7e

Every solution names the midline, amplitude and period before building or reading an equation. Compare these three values with your own, and check each model at one point.

Standards: F-TF.B.5, F-IF.C.7e · **Level:** Grade 11 Algebra 2

Problem 1 Amplitude and period of sine

In $y = A \sin(Bx)$, $A = 4$ and $B = 1$. **Amplitude 4, period 2π .**

Problem 2 Period of a compressed cosine

Period = $2\pi/3$ since $B = 3$, and the amplitude is 1. **Period $2\pi/3$, amplitude 1.**

Problem 3 Reflected sine

The amplitude is $|-2| = 2$ and the period is 2π . The negative sign reflects the graph across the x-axis, so it dips first. **Amplitude 2, period 2π , reflected.**

Problem 4 Vertical shift

The midline is $y = 5$. The maximum is $5 + 1 = 6$ and the minimum is $5 - 1 = 4$. **Midline $y = 5$; max 6; min 4.**

Problem 5 Range of a cosine function

Midline -1 and amplitude 3: max = $-1 + 3 = 2$ and min = $-1 - 3 = -4$. **Max 2, min -4 , range $-4 \leq y \leq 2$.**

Problem 6 Period with π in the coefficient

Here $B = \pi/4$, so the period is $2\pi \div (\pi/4) = 8$. **Period 8.**

Problem 7 Write an equation from a graph

The maximum is 3, so $A = 3$. One full wave takes π (zeros at $0, \pi/2, \pi$), so $B = 2\pi/\pi = 2$. **$y = 3 \sin 2x$.**

Problem 8 Phase shift to the right

The graph shifts right $\pi/3$. The upward midline crossing moves from 0 to $\pi/3$. The maximum moves from $\pi/2$ to $\pi/2 + \pi/3 = 5\pi/6$. **Shift right $\pi/3$; crossing at $\pi/3$; max at $5\pi/6$.**

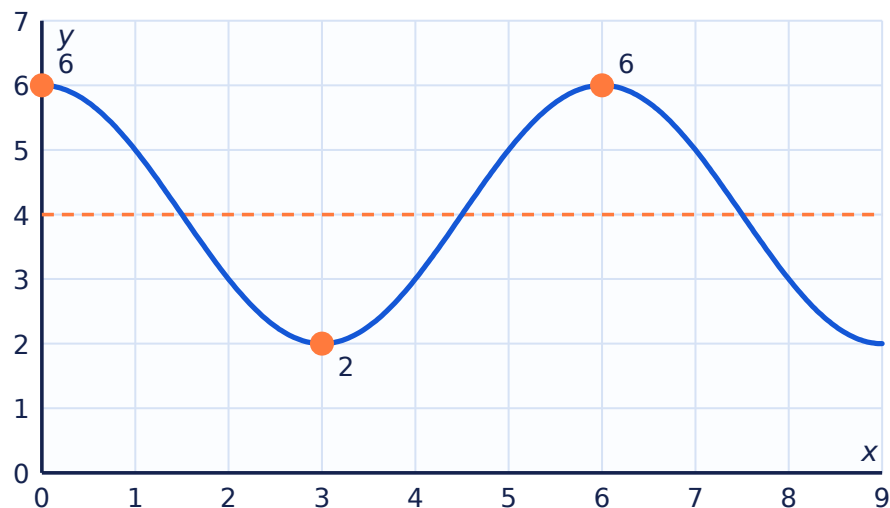
Problem 9 Shift left and down

Write $x + \pi/2 = x - (-\pi/2)$, so the shift is $\pi/2$ to the left. Amplitude 2, midline $y = -3$.

Amplitude 2, shift left $\pi/2$, midline $y = -3$.

Problem 10 Read features from a graph

Maximum 6 at $x = 0$ and 6; minimum 2 at $x = 3$ and 9. Midline = $(6 + 2)/2 = 4$, amplitude = $(6 - 2)/2 = 2$, and the period is 6 (peaks at 0 and 6). **Max 6, min 2, midline 4, amplitude 2, period 6.**



Answer figure: cosine wave with maximum 6 at x equals 0 and 6, minimum 2 at x equals 3 and 9, and midline 4

Problem 11 Cosine from a description

$B = 2\pi/12 = \pi/6$. $y = 4 \cos(\pi x/6) + 6$. Check: at $x = 0$, $y = 4 + 6 = 10$, a maximum.

Problem 12 Sine from a description

$B = 2\pi/10 = \pi/5$. $y = 2.5 \sin(\pi x/5)$. Check: at $x = 2.5$ it equals $2.5 \sin(\pi/2) = 2.5$.

Problem 13 All features at once

Factor the inside: $2x - \pi/2 = 2(x - \pi/4)$. Amplitude 3, period $2\pi/2 = \pi$, shift $\pi/4$ to the right.

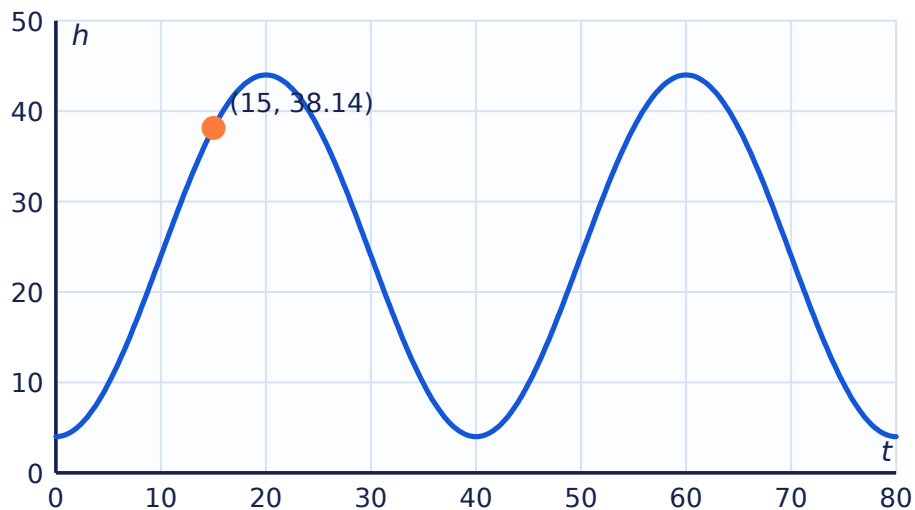
Amplitude 3, period π , phase shift $\pi/4$.

Problem 14 Locate maximum points

A maximum occurs when $4x = \pi/2 + 2\pi k$, so $x = \pi/8 + \pi k/2$. For $k = 0$ and 1 this gives $\pi/8$ and $5\pi/8$; $k = 2$ gives $9\pi/8$, which is beyond π . **$x = \pi/8$ and $5\pi/8$** , each with $y = 5$.

Problem 15 Ferris wheel model

$h(15) = -20 \cos(3\pi/4) + 24 = -20(-0.70711) + 24 = 38.1421$. **≈ 38.14 feet.**



Answer figure: height curve with the point (15, 38.14)

Problem 16 Tide model

Midline = $(7.5 + 1.5)/2 = 4.5$, amplitude 3, period 12, so $B = \pi/6$. $d(t) = 3 \cos(\pi t/6) + 4.5$. $d(2) = 3 \cos(\pi/3) + 4.5 = 6$. The depth is 4.5 when $\cos(\pi t/6) = 0$, so $\pi t/6 = \pi/2$ and $t = 3$. **$d(t) = 3 \cos(\pi t/6) + 4.5$; 6 feet; at $t = 3$ hours.**

Problem 17 Daylight hours

Midline 12.1 and amplitude 2.8: longest 14.9 hours, shortest 9.3 hours. The period is $2\pi \div (2\pi/365) = 365$ days. $D(120) = 2.8 \sin(2\pi \cdot 40/365) + 12.1 = 2.8(0.6362) + 12.1 = 13.879$. **Longest 14.9 h, shortest 9.3 h, period 365 days, $D(120) \approx 13.9$ h.**

Problem 18 Daily temperature

Max = $68 + 14 = 82^\circ\text{F}$ and min = $68 - 14 = 54^\circ\text{F}$. The sine peaks when $\pi(t - 9)/12 = \pi/2$, so $t - 9 = 6$ and $t = 15$ (3 PM). $T(12) = 14 \sin(\pi/4) + 68 = 77.899$. **82°F at 3 PM; 54°F ; $T(12) \approx 77.9^\circ\text{F}$.**

Problem 19 Show two forms are equal

At $x = 0$: $\cos 0 = 1$ and $\sin(\pi/2) = 1$. At $\pi/2$: $\cos = 0$ and $\sin(\pi) = 0$. At π : $\cos \pi = -1$ and $\sin(3\pi/2) = -1$. Shifting the sine graph left $\pi/2$ moves its maximum from $\pi/2$ to 0, matching cosine. **The two functions are identical.**

Problem 20 Find a model from extreme values

Midline $(10 + (-2))/2 = 4$, amplitude 6. Half a period is $4 - 1 = 3$, so the period is 6 and $B = \pi/3$. The maximum is at $t = 1$, so $y = 6 \cos(\pi(t - 1)/3) + 4$. Check: at $t = 4$, $6 \cos \pi + 4 = -2$. **$y = 6 \cos(\pi(t - 1)/3) + 4$.**