

Combining and Inverting Functions

Grade 11 – Algebra 2 math · Lesson · Standards: F-BF.A.1b, F-BF.A.1c, F-BF.B.4a

Quick answer: Functions combine by adding, subtracting, multiplying, or composing; in $f(g(x))$ g feeds f . An inverse f^{-1} undoes f : swap x and y , solve for y , and verify $f(f^{-1}(x)) = x$. Its graph reflects f across $y = x$.

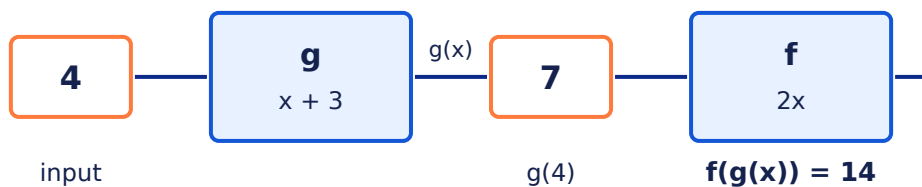
Standards: F-BF.A.1b, F-BF.A.1c, F-BF.B.4a · **Level:** Grade 11 Algebra 2

How do you combine functions with arithmetic?

Two functions can be added, subtracted, or multiplied by working with their outputs: $(f + g)(x) = f(x) + g(x)$, $(f - g)(x) = f(x) - g(x)$, and $(f \cdot g)(x) = f(x) \cdot g(x)$. The domain is the set of inputs that both functions accept.

What is function composition?

In the **composition** $(f \circ g)(x) = f(g(x))$, the output of g becomes the input of f . Work from the inside out.



With $g(x) = x + 3$ and $f(x) = 2x$, the input 4 gives $g(4) = 7$ and $f(7) = 14$.

Common mistake

Composition is usually not commutative: $f(g(x))$ and $g(f(x))$ are different functions. For $f(x) = 2x$ and $g(x) = x + 3$, $f(g(x)) = 2x + 6$ but $g(f(x)) = 2x + 3$.

How do you find an inverse function?

Inverse function

Functions f and f^{-1} are **inverses** when $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. The inverse undoes the function by swapping inputs and outputs, so (a, b) on f becomes (b, a) on f^{-1} . Only one-to-one functions, which pass the horizontal line test, have inverse functions.

Method: find f^{-1}

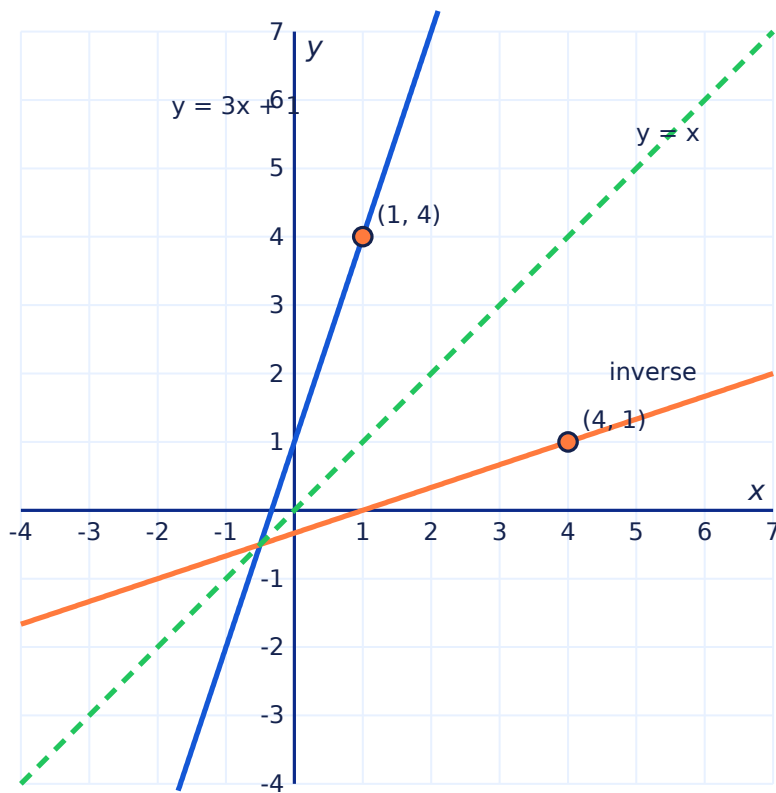
1. Write $y = f(x)$.
2. Swap x and y .
3. Solve the new equation for y .
4. Write $f^{-1}(x) =$ the result, and verify by composition.

Worked example

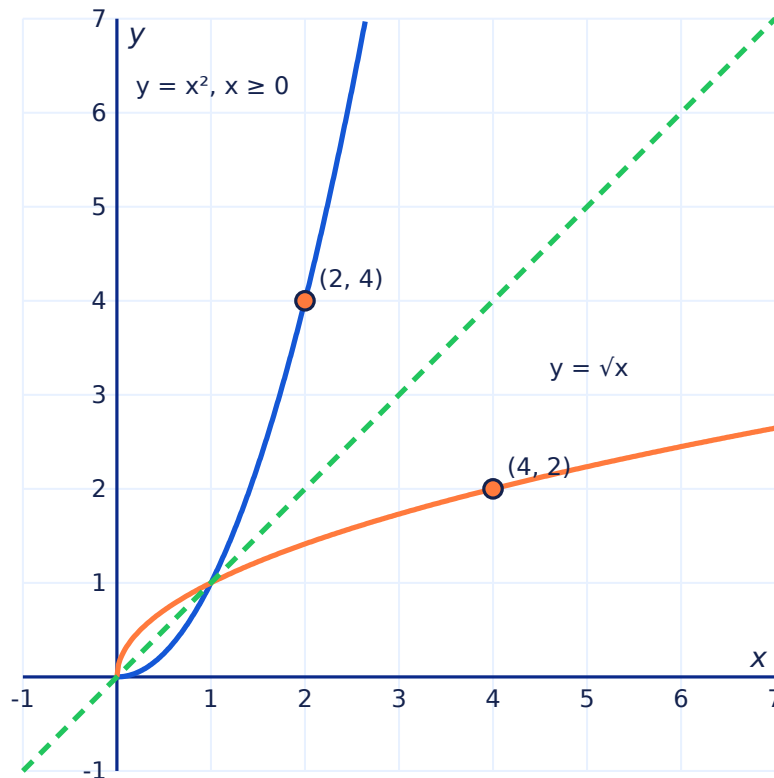
For $f(x) = 3x + 1$: $y = 3x + 1$; swap to $x = 3y + 1$; solve $y = \frac{(x-1)}{3}$. So $f^{-1}(x) = \frac{(x-1)}{3}$. Check: $f(\frac{(x-1)}{3}) = 3 \cdot \frac{(x-1)}{3} + 1 = x \checkmark$.

What do the graphs of inverses look like?

The graph of f^{-1} is the reflection of the graph of f across the line $y = x$.



$(1, 4)$ on f matches $(4, 1)$ on f^{-1} .



$y = x^2$ for $x \geq 0$ and $y = \sqrt{x}$ are inverses; the restriction makes x^2 one-to-one.

Mathbot's tip

If a function fails the horizontal line test, restrict its domain to one monotone piece, such as $x \geq 0$ for x^2 , and then invert.

Key points to remember

- $(f + g)(x)$, $(f - g)(x)$, and $(f \cdot g)(x)$ combine the outputs of f and g .
- Composition $f(g(x))$ works inside-out and is not usually commutative.
- Find an inverse by swapping x and y and solving for y .
- Inverses satisfy $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
- The graphs of f and f^{-1} are mirror images across the line $y = x$.

Combining and Inverting Functions: frequently asked questions

What does $f(g(x))$ mean?

It means apply g first and then apply f to the result. Substitute the whole expression for $g(x)$ wherever x appears in f .

Is $f(g(x))$ always equal to $g(f(x))$?

No. For $f(x) = 2x$ and $g(x) = x + 3$, $f(g(x)) = 2x + 6$ but $g(f(x)) = 2x + 3$. Order matters in most compositions.

How do you check that two functions are inverses?

Compose them in both orders. If $f(g(x)) = x$ and $g(f(x)) = x$ for all inputs in the domains, then f and g are inverses.

Which functions have an inverse function?

Only one-to-one functions. A graph passes the horizontal line test when every horizontal line meets it at most once, which means each output comes from exactly one input.

How do you graph an inverse function?

Reflect the graph of f across the line $y = x$, or swap the coordinates of every point: (a, b) on f becomes (b, a) on the inverse.