

Combining and Inverting Functions

Grade 11 – Algebra 2 math · Test Answer Key · Standards: F-BF.A.1b, F-BF.A.1c, F-BF.B.4a

Each solution shows the substitution or the swap-and-solve steps and a scoring guide. Accept any equivalent form of an expression, and give partial credit for a correct method with an arithmetic slip.

Standards: F-BF.A.1b, F-BF.A.1c, F-BF.B.4a · **Level:** Grade 11 Algebra 2

Question 1 Combining functions (2 points)

$$(h + k)(x) = x^2 - 3x + 2x + 7 = \mathbf{x^2 - x + 7}.$$

$$h(-2) = 4 + 6 = 10 \text{ and } k(-2) = 3, \text{ so } (h \cdot k)(-2) = \mathbf{30}.$$

Scoring: 1 point each.

Question 2 Composition (3 points)

$$h(k(x)) = 3(x^2 + 1) - 2 = \mathbf{3x^2 + 1}.$$

$$k(h(x)) = (3x - 2)^2 + 1 = \mathbf{9x^2 - 12x + 5}.$$

$$k(2) = 5 \text{ and } h(5) = 13, \text{ so } h(k(2)) = \mathbf{13}.$$

Scoring: 1 point each.

Question 3 Find an inverse (3 points)

$$y = \frac{(3x - 8)}{5}. \text{ Swap: } x = \frac{(3y - 8)}{5}, \text{ so } 5x = 3y - 8 \text{ and } y = \frac{(5x + 8)}{3}.$$

$$\mathbf{h^{-1}(x) = \frac{(5x + 8)}{3}}$$

$$\text{Check: } h(h^{-1}(x)) = \frac{(3 \cdot \frac{(5x + 8)}{3} - 8)}{5} = \frac{(5x + 8 - 8)}{5} = x \checkmark.$$

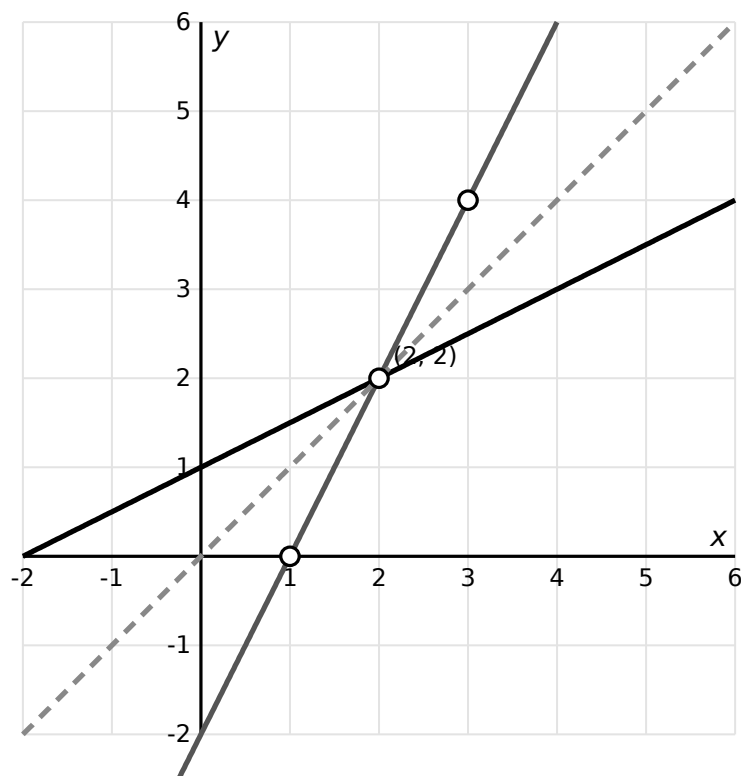
Scoring: 1 point for swapping, 1 for solving, 1 for the verification.

Question 4 Inverse from a graph (3 points)

a) Swap coordinates: **(1, 0), (2, 2), (3, 4).**

b) $h(x) = \frac{x}{2} + 1$. Swap: $x = \frac{y}{2} + 1$, so $y = 2x - 2$. **$h^{-1}(x) = 2x - 2$**

c) The point on the line $y = x$: **(2, 2)**.



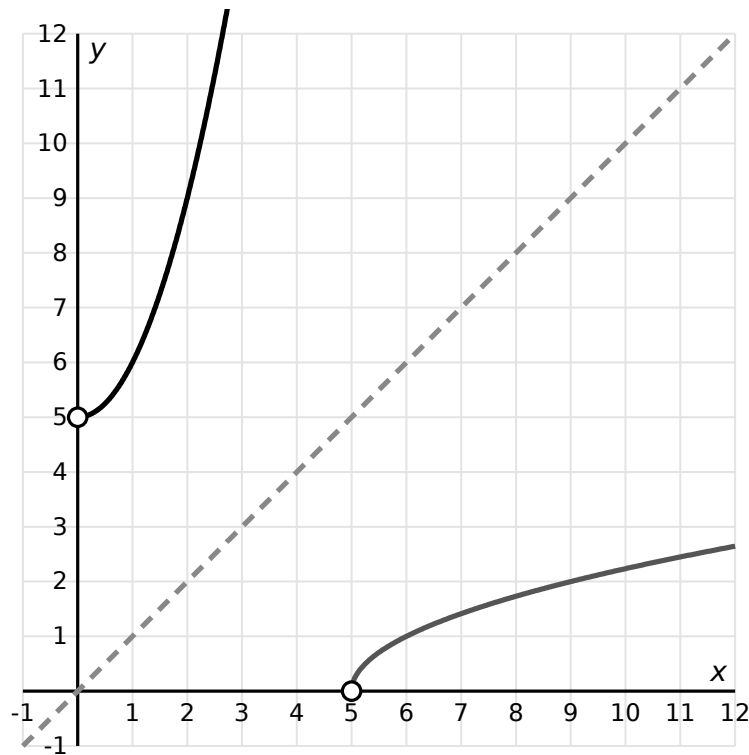
h , h^{-1} , and $y = x$.

Scoring: 1 point per part.

Question 5 Restricted domain (3 points)

a) $y = x^2 + 5$. Swap: $x = y^2 + 5$, so $y = \sqrt{x - 5}$ (positive root). **$h^{-1}(x) = \sqrt{x - 5}$**

b) The outputs of h are $y \geq 5$, so the domain of h^{-1} is **$x \geq 5$** , and its range is **$y \geq 0$** .



h and its inverse.

Scoring: 1 point for swapping, 1 for the positive root, 1 for the domain and range.

Question 6 Domain of a composition (2 points)

$h(k(x)) = \sqrt{x^2 - 7}$, which requires $x^2 \geq 7$: domain $x \leq -\sqrt{7}$ or $x \geq \sqrt{7}$.

$k(h(x)) = (\sqrt{x - 7})^2 = x - 7$, which requires $x - 7 \geq 0$: domain $x \geq 7$.

Scoring: 1 point each, including the domain.

Question 7 Sale and coupon (2 points)

Sale first: $S(48) = 36$, then $K(36) = 28$.

Coupon first: $K(48) = 40$, then $S(40) = 30$.

Sale first costs \$28, which is \$2 cheaper than \$30

Scoring: 1 point for both prices, 1 point for the conclusion.

Question 8 Verify inverses (2 points)

$h(k(x)) = (\sqrt[3]{x + 1})^3 - 1 = x + 1 - 1 = x$.

$$k(h(x)) = \sqrt[3]{((x^3 - 1) + 1)} = \sqrt[3]{(x^3)} = x.$$

Both compositions give x , so h and k are inverses.

Scoring: 1 point for each composition.