

Combining and Inverting Functions

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: F-BF.A.1b, F-BF.A.1c, F-BF.B.4a

Each solution shows the substitution or swap-and-solve steps and the final answer in bold.
Check inverses by composing in both orders.

Standards: F-BF.A.1b, F-BF.A.1c, F-BF.B.4a · **Level:** Grade 11 Algebra 2

Problem 1 Adding, subtracting, and multiplying functions

$$(h + k)(x) = 5x + 2 + x^2 - 6 = \mathbf{x^2 + 5x - 4}.$$

$$(h - k)(x) = 5x + 2 - x^2 + 6 = \mathbf{-x^2 + 5x + 8}.$$

$$(h \cdot k)(x) = (5x + 2)(x^2 - 6) = \mathbf{5x^3 + 2x^2 - 30x - 12}.$$

Problem 2 Evaluating combined functions

$$h(3) = 17 \text{ and } k(3) = 3, \text{ so } (h + k)(3) = 20.$$

$$h(-2) = -8 \text{ and } k(-2) = -2, \text{ so } (h \cdot k)(-2) = 16.$$

20 and 16

Problem 3 Composition at a number

$$k(5) = 11, \text{ so } h(k(5)) = h(11) = 37.$$

$$h(5) = 13, \text{ so } k(h(5)) = k(13) = 19.$$

37 and 19

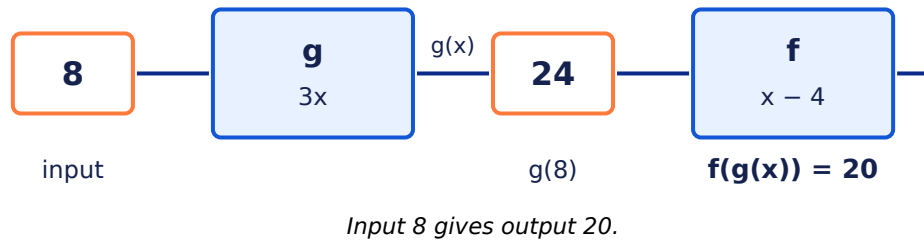
Problem 4 Composition as an expression

$$h(k(x)) = (x - 5)^2 + 3 = x^2 - 10x + 25 + 3 = \mathbf{x^2 - 10x + 28}.$$

$$k(h(x)) = (x^2 + 3) - 5 = \mathbf{x^2 - 2}.$$

Problem 5 A function machine

$k(8) = 24$, then $h(24) = 24 - 4 = 20$.



20

Problem 6 A linear inverse

$y = 4x + 9$. Swap: $x = 4y + 9$. Solve: $4y = x - 9$, so $y = \frac{(x-9)}{4}$.

$$h^{-1}(x) = \frac{(x-9)}{4}$$

Problem 7 An inverse with a fraction

$y = \frac{(5x-2)}{4}$. Swap: $x = \frac{(5y-2)}{4}$. Then $4x = 5y - 2$, so $y = \frac{(4x+2)}{5}$.

$$h^{-1}(x) = \frac{(4x+2)}{5}$$

Problem 8 Verifying inverses

$$h(k(x)) = 6 \cdot \frac{(x-1)}{6} + 1 = x - 1 + 1 = x.$$

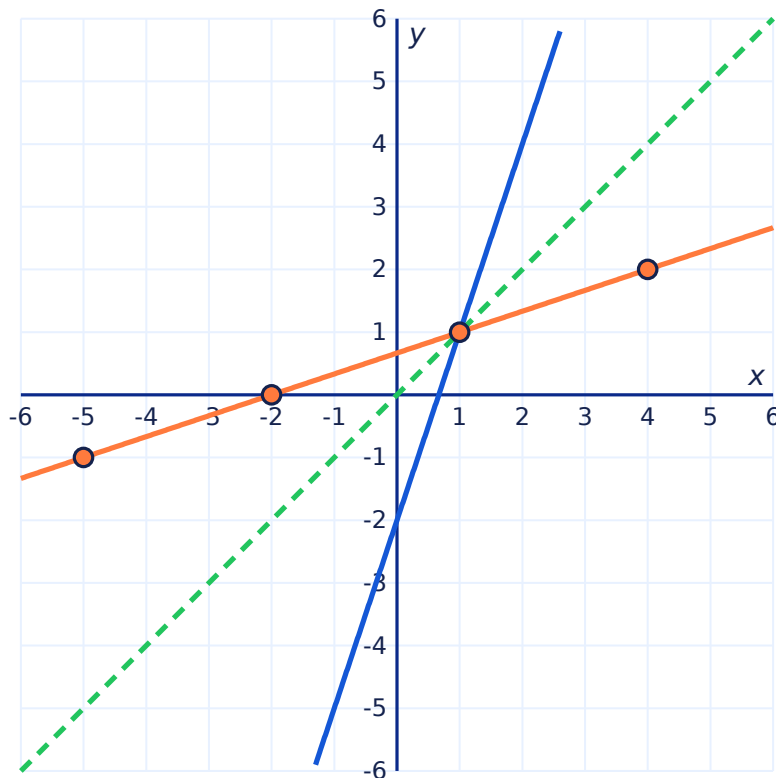
$$k(h(x)) = \frac{((6x+1)-1)}{6} = \frac{6x}{6} = x.$$

Both compositions equal x , so h and k are inverses.

Problem 9 Reflecting a graph

Swap the coordinates: $(-5, -1)$, $(-2, 0)$, $(1, 1)$, $(4, 2)$.

Equation: swap $x = 3y - 2$, so $y = \frac{(x+2)}{3}$.



h^{-1} is the mirror image of h .

$(-5, -1), (-2, 0), (1, 1), (4, 2); h^{-1}(x) = \sqrt[3]{x+2}$

Problem 10 A cube root inverse

$y = x^3 + 6$. Swap: $x = y^3 + 6$, so $y^3 = x - 6$ and $y = \sqrt[3]{x - 6}$.

$h^{-1}(14) = \sqrt[3]{8} = 2$.

$h^{-1}(x) = \sqrt[3]{x - 6}; h^{-1}(14) = 2$

Problem 11 A restricted domain

$y = x^2 - 7$. Swap: $x = y^2 - 7$, so $y^2 = x + 7$ and $y = \sqrt{x + 7}$ (positive root because the original inputs are $x \geq 0$).

The outputs of h are $y \geq -7$, so the domain of h^{-1} is $x \geq -7$.

$h^{-1}(9) = \sqrt{16} = 4$ and $h(4) = 16 - 7 = 9 \checkmark$.

$h^{-1}(x) = \sqrt{x + 7}, x \geq -7$

Problem 12 Inverse of a square root function

$y = \sqrt{x + 3}$, with $x \geq -3$ and $y \geq 0$. Swap: $x = \sqrt{y + 3}$, so $x^2 = y + 3$ and $y = x^2 - 3$.

The outputs of h are $y \geq 0$, so the inverse has domain $x \geq 0$.

$$h^{-1}(x) = x^2 - 3, x \geq 0$$

Problem 13 Domain of a composition

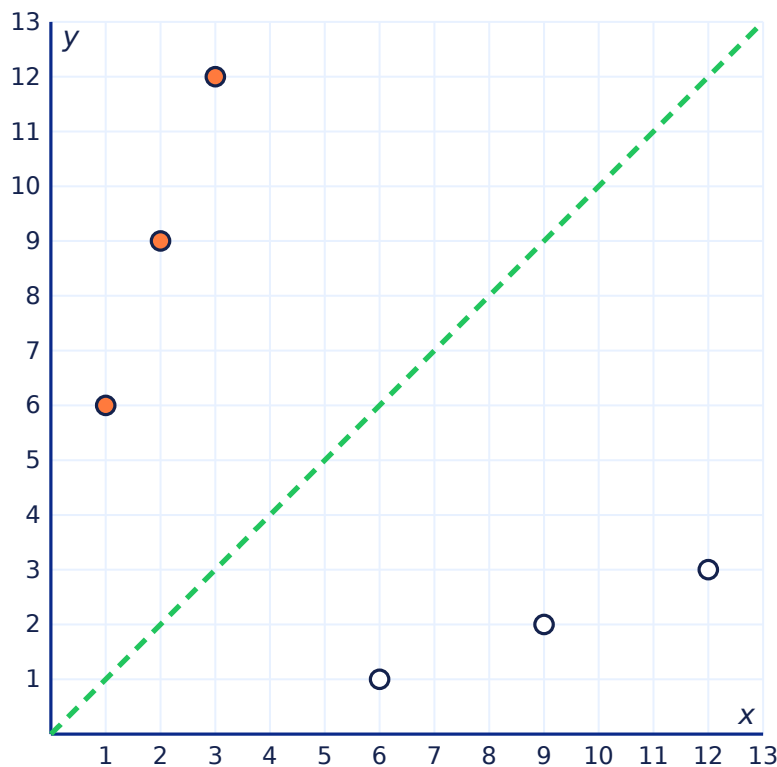
$h(k(x)) = \sqrt{x + 8}$, which needs $x + 8 \geq 0$: domain $x \geq -8$.

$k(h(x)) = \sqrt{x + 8}$, which needs $x \geq 0$: domain $x \geq 0$.

$$\sqrt{x + 8}, x \geq -8; \sqrt{x + 8}, x \geq 0$$

Problem 14 Inverse of a table

Swap each pair: (6, 1), (9, 2), (12, 3). So $h^{-1}(12) = 3$.



Each filled point reflects to an open point across $y = x$.

$$(6, 1), (9, 2), (12, 3); h^{-1}(12) = 3$$

Problem 15 A rational inverse

$y = \frac{(5x+3)}{(x-2)}$. Swap: $x = \frac{(5y+3)}{(y-2)}$, so $xy - 2x = 5y + 3$ and $y(x - 5) = 2x + 3$.

$$h^{-1}(x) = \frac{(2x+3)}{(x-5)}.$$

Check: $h(h^{-1}(x)) = \frac{(5(2x+3)/(x-5) + 3)}{((2x+3)/(x-5) - 2)} = \frac{((10x+15) + 3(x-5))}{((2x+3) - 2(x-5))} = \frac{13x}{13} = x \checkmark$.

$$h^{-1}(x) = \frac{(2x+3)}{(x-5)}$$

Problem 16 Converting temperatures

$y = \frac{9}{5}x + 32$. Swap: $x = \frac{5}{9}y + 32$, so $y = \frac{5}{9}(x - 32)$.

$$F^{-1}(98.6) = \frac{5}{9}(66.6) = 37.$$

Check: $F(F^{-1}(x)) = \frac{9}{5} \cdot \frac{5}{9}(x - 32) + 32 = x \checkmark$.

$$C(h) = \frac{5}{9}(h - 32); 98.6^{\circ}\text{F} = 37^{\circ}\text{C}$$

Problem 17 Order of discounts

Coupon first: $C(60) = 50$, then $D(50) = 40$.

Sale first: $D(60) = 48$, then $C(48) = 38$.

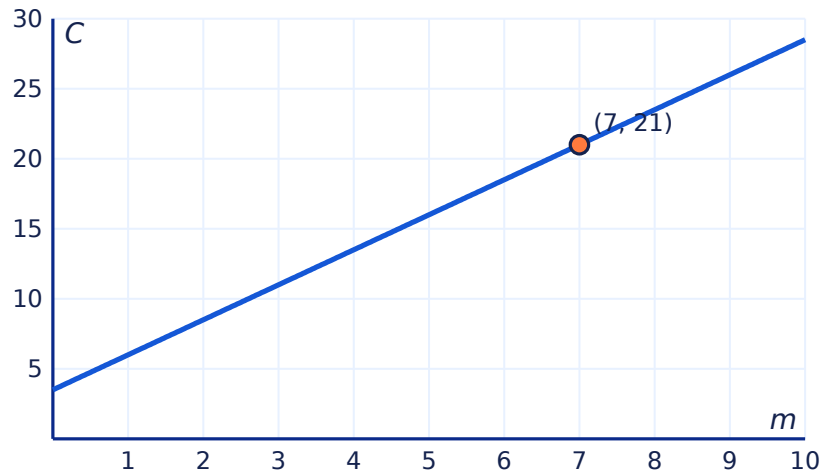
In general $D(C(p)) = 0.8p - 8$ and $C(D(p)) = 0.8p - 10$, so the second order is always \$2 cheaper.

\$40 and \$38; apply the sale first

Problem 18 Taxi fare and miles

$y = 2.5x + 3.5$. Swap: $x = \frac{(y-3.5)}{2.5}$.

For \$21: $\frac{(21-3.5)}{2.5} = 7$.



A \$21 fare is 7 miles.

$$m = \frac{(C - 3.5)}{2.5}; \text{ 7 miles}$$

Problem 19 Making a function invertible

Complete the square: $h(x) = (x - 3)^2 - 4$. On $x \geq 3$ the outputs are $y \geq -4$.

Swap: $x = (y - 3)^2 - 4$, so $(y - 3)^2 = x + 4$ and $y - 3 = \sqrt{x + 4}$ (the positive root, since $y \geq 3$).

$$h^{-1}(x) = 3 + \sqrt{x + 4}; \text{ domain } x \geq -4; \text{ range } y \geq 3$$

Problem 20 When do two compositions agree?

$$h(k(x)) = 4(x^2 + 3) - 1 = 4x^2 + 11.$$

$$k(h(x)) = (4x - 1)^2 + 3 = 16x^2 - 8x + 4.$$

$$\text{Set equal: } 4x^2 + 11 = 16x^2 - 8x + 4, \text{ so } 12x^2 - 8x - 7 = 0.$$

$$x = \frac{(8 \pm \sqrt{64 + 336})}{24} = \frac{(8 \pm 20)}{24}, \text{ so } x = \frac{7}{6} \text{ or } x = -\frac{1}{2}.$$

$$x = \frac{7}{6} \text{ or } x = -\frac{1}{2}$$