

Combining and Inverting Functions

Grade 11 – Algebra 2 math · Worksheet · Standards: F-BF.A.1b, F-BF.A.1c, F-BF.B.4a

Name: _____ Date: _____ Class: _____

These problems cover operations on functions, composition, inverse functions, and the reflection of a graph across the line $y = x$. They follow Common Core standards F-BF.A.1b, F-BF.A.1c, and F-BF.B.4a.

Early problems build combined functions and evaluate compositions; later ones invert linear, root, rational, and restricted functions and use inverses in real contexts such as temperature, discounts, and taxi fares.

Standards: F-BF.A.1b, F-BF.A.1c, F-BF.B.4a · **Level:** Grade 11 Algebra 2

Problem 1 Adding, subtracting, and multiplying functions (easy)

For $h(x) = 5x + 2$ and $k(x) = x^2 - 6$, find $(h + k)(x)$, $(h - k)(x)$, and $(h \cdot k)(x)$.

Problem 2 Evaluating combined functions (easy)

Using the same h and k , find $(h + k)(3)$ and $(h \cdot k)(-2)$.

Problem 3 Composition at a number (easy)

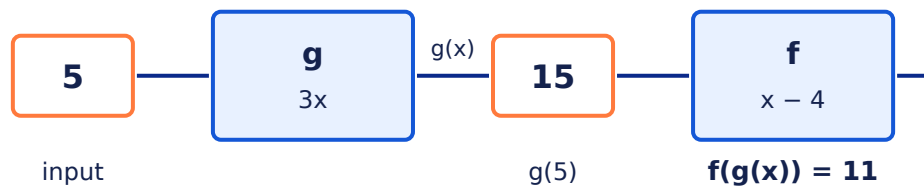
Let $h(x) = 4x - 7$ and $k(x) = x + 6$. Find $h(k(5))$ and $k(h(5))$.

Problem 4 Composition as an expression (easy)

For $h(x) = x^2 + 3$ and $k(x) = x - 5$, find $h(k(x))$ and $k(h(x))$.

Problem 5 A function machine (easy)

An input is processed by $k(x) = 3x$ and then by $h(x) = x - 4$. The figure shows the machine for the input 5. Find the output for the input 8.



Input 5 gives output 11.

Problem 6 A linear inverse (medium)

Determine the inverse of $h(x) = 4x + 9$.

Problem 7 An inverse with a fraction (medium)

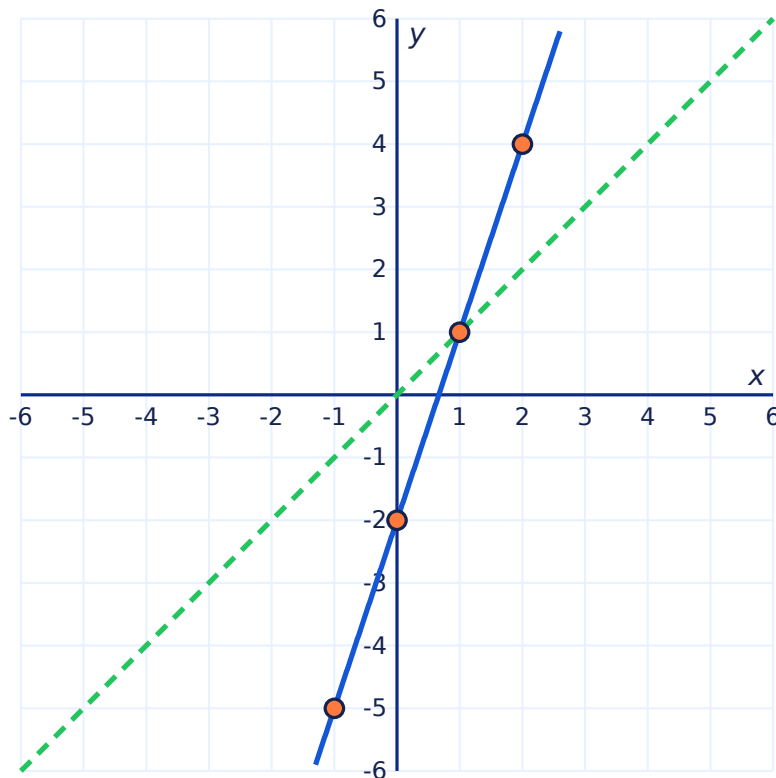
Determine the inverse of $h(x) = \frac{(5x - 2)}{4}$.

Problem 8 Verifying inverses (medium)

Show by composition that $h(x) = 6x + 1$ and $k(x) = \frac{(x - 1)}{6}$ are inverses.

Problem 9 Reflecting a graph (medium)

The graph of $h(x) = 3x - 2$ and the line $y = x$ are shown. Reflect the points $(-1, -5)$, $(0, -2)$, $(1, 1)$, and $(2, 4)$ of h across $y = x$, then write the equation of h^{-1} .



Graph of h and the line $y = x$.

Problem 10 A cube root inverse (medium)

Determine the inverse of $h(x) = x^3 + 6$ and evaluate $h^{-1}(14)$.

Problem 11 A restricted domain (medium)

The function $h(x) = x^2 - 7$ is restricted to $x \geq 0$. Find h^{-1} , its domain, and verify $h(h^{-1}(9)) = 9$.

Problem 12 Inverse of a square root function (medium)

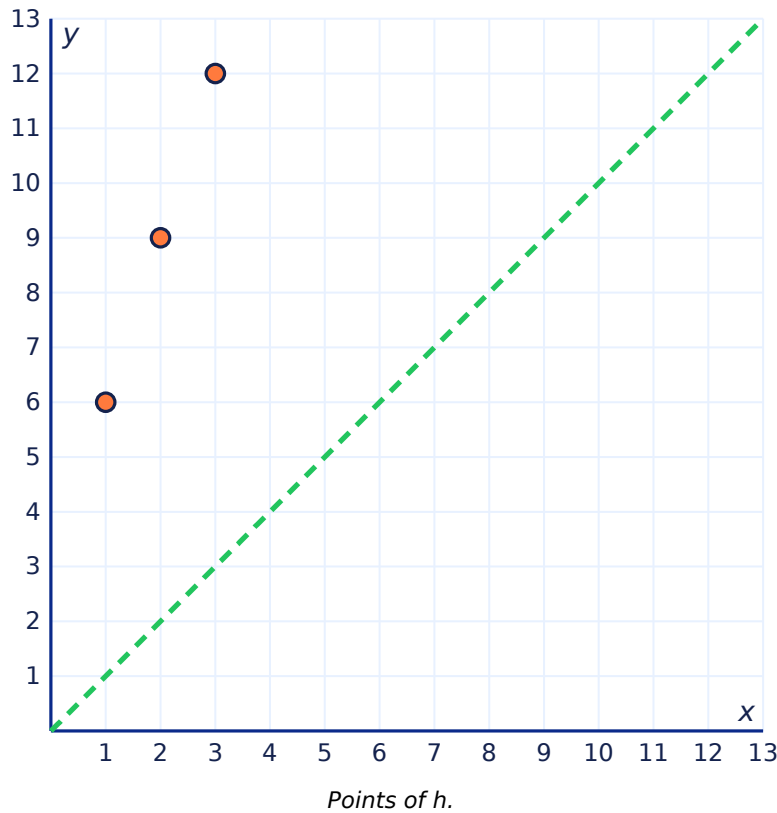
Determine the inverse of $h(x) = \sqrt{x + 3}$, and state the domain of h^{-1} .

Problem 13 Domain of a composition (medium)

For $h(x) = \sqrt{x}$ and $k(x) = x + 8$, write $h(k(x))$ and $k(h(x))$, and give the domain of each.

Problem 14 Inverse of a table (medium)

A function h is given by the points $(1, 6)$, $(2, 9)$, and $(3, 12)$. Write the points of h^{-1} , find $h^{-1}(12)$, and use the figure to explain the relationship.



Problem 15 A rational inverse (hard)

Determine the inverse of $h(x) = \frac{(5x + 3)}{(x - 2)}$ and check it by computing $h(h^{-1}(x))$.

Problem 16 Converting temperatures (hard)

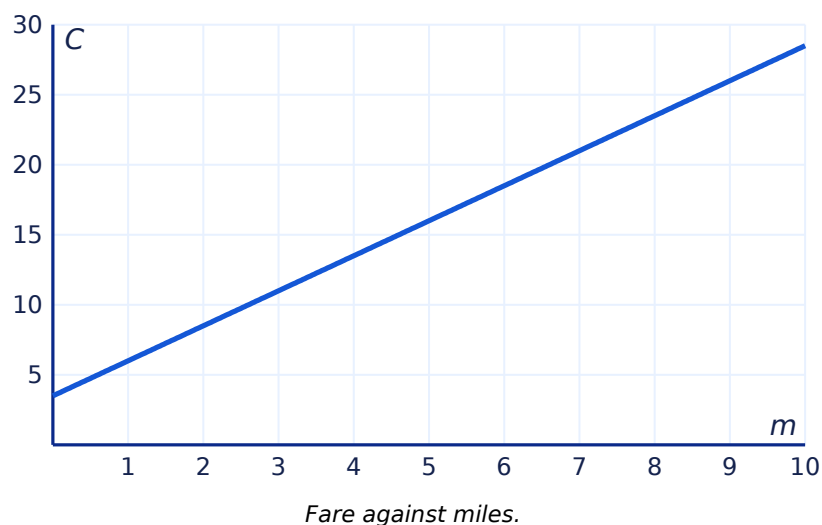
A food scientist converts Celsius to Fahrenheit with $F(c) = \frac{9}{5}c + 32$. Find the inverse function that converts Fahrenheit to Celsius, use it for 98.6°F , and verify by composition.

Problem 17 Order of discounts (hard)

A shopper has a \$10-off coupon, $C(p) = p - 10$, and a 20%–off sale, $D(p) = 0.8p$. For a \$60 jacket, find $D(C(60))$ and $C(D(60))$, and decide which order saves more money.

Problem 18 Taxi fare and miles (hard)

A taxi fare is $C(m) = 2.5m + 3.5$ dollars for m miles. Find the inverse function that gives miles from the fare, and use it to find the distance for a \$21 fare. The figure shows the fare graph.



Problem 19 Making a function invertible (hard)

The function $h(x) = x^2 - 6x + 5$ fails the horizontal line test. Restrict the domain to $x \geq 3$, find h^{-1} , and state its domain and range.

Problem 20 When do two compositions agree? (hard)

Let $h(x) = 4x - 1$ and $k(x) = x^2 + 3$. There are exactly 2 values of x for which $h(k(x)) = k(h(x))$; find both.