

Logarithms: The Inverse of Exponents

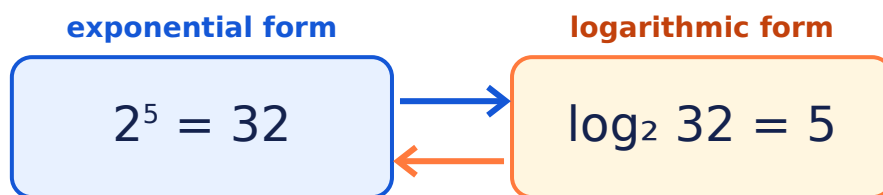
Grade 11 – Algebra 2 math · Lesson · Standards: F-LE.A.4, F-BF.B.4a, F-IF.C.7e

Quick answer: A logarithm is the exponent that a base must be raised to in order to produce a given number. For example, log base 2 of 32 equals 5 because 2 raised to the 5th power is 32. Logarithms are the inverse of exponential functions, and their graphs are mirror images across the line $y = x$.

Standards: F-LE.A.4, F-BF.B.4a, F-IF.C.7e · **Level:** Grade 11 Algebra 2

What is a logarithm?

A logarithm answers the question “which exponent do I need?” Because $2^5 = 32$, the logarithm of 32 in base 2 is 5.



base 2 stays the base; the exponent 5 becomes the answer

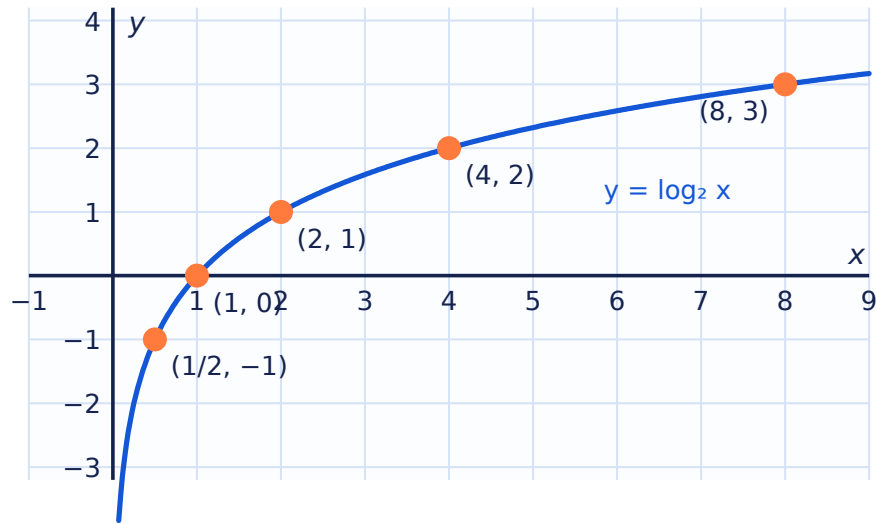
Both statements say the same thing: 2 raised to 5 gives 32.

Definition

For $b > 0$, $b \neq 1$ and $x > 0$: $\log_b x = y$ exactly when $b^y = x$. A log with no written base means base 10 (log), and ln means base $e \approx 2.718$.

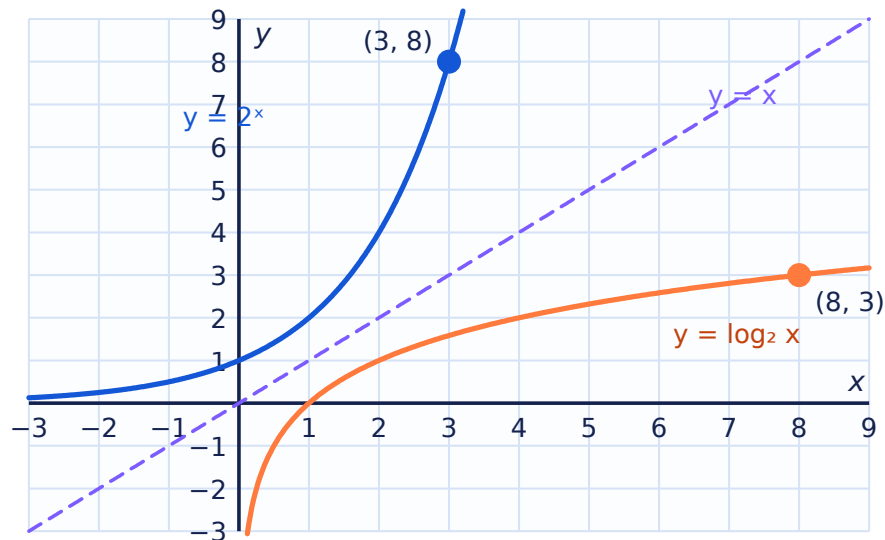
What does the graph of a logarithmic function look like?

The graph of $y = \log_2 x$ passes through (1, 0) because $b^0 = 1$ for every base. It rises slowly, and the y-axis is a vertical asymptote. The domain is $x > 0$ and the range is all real numbers.



The graph of $y = \log_2 x$ rises slowly and never touches the y-axis.

Since logarithms undo exponentials, the two graphs are mirror images over the line $y = x$.



The exponential and logarithmic graphs are mirror images over the dashed line $y = x$.

Which properties make logarithms useful?

Properties of logarithms

- Product: $\log_b(MN) = \log_b M + \log_b N$
- Quotient: $\log_b(\frac{M}{N}) = \log_b M - \log_b N$
- Power: $\log_b(M^p) = p \cdot \log_b M$
- Change of base: $\log_b x = \frac{(\ln x)}{(\ln b)}$

How do you expand a logarithm?

Method

1. Write the argument as a product, quotient and powers.
2. Split products into sums and quotients into differences.
3. Move each exponent in front as a multiplier.

Example

Expand $\log_5(25a^3/c)$.

$$\log_5 25 + \log_5 a^3 - \log_5 c = 2 + 3 \log_5 a - \log_5 c.$$

Common mistake

$\log_b(M + N)$ is not $\log_b M + \log_b N$. There is no rule for the log of a sum.

Where are logarithms used?

Logarithmic scales squeeze huge ranges into small numbers: pH = $-\log[\text{H}^+]$, sound level $L = 10 \log(I/I_0)$ in decibels, and earthquake magnitude. Adding 1 to pH means ten times fewer hydrogen ions.

Mathbot's tip

Say it as a question: “ $\log_3 81$ means 3 to what power gives 81?” The answer, 4, is the log.

Key points to remember

- $\log_b x = y$ means $b^y = x$, with $b > 0$, $b \neq 1$ and $x > 0$.
- The graph of $y = \log_b x$ passes through (1, 0) and has the y-axis as a vertical asymptote.
- Product, quotient and power rules turn multiplication into addition, division into subtraction and exponents into multipliers.
- Change of base: $\log_b x = \ln x \div \ln b$, which lets a calculator find any logarithm.
- log means base 10 and ln means base $e \approx 2.718$.

Logarithms: The Inverse of Exponents: frequently asked questions

What is the difference between log and ln?

The notation log means base 10, and ln means base e, where e is about 2.718. Both follow the same properties. Use log for powers of 10 and ln when e appears, as in continuous growth.

Why can you not take the log of a negative number?

A positive base raised to any real power always gives a positive result. No exponent can produce a

negative number or zero, so the argument of a logarithm must be greater than 0.

How do you find log base 3 of 50 on a calculator?

Use change of base: $\log_3 50 = \ln 50 \div \ln 3 \approx 3.912 \div 1.099 \approx 3.56$. Calculators have only log and ln keys, so every other base is converted this way.

Why does every logarithmic graph pass through (1, 0)?

Any base raised to the power 0 equals 1, so $\log_b 1 = 0$ for every valid base b. That is why the x-intercept of $y = \log_b x$ is always 1.

What does $\log(AB) = \log A + \log B$ really say?

Multiplying two numbers adds their exponents when written with the same base. Since a logarithm is an exponent, the log of a product is the sum of the logs. This is why logs once replaced slide-rule multiplication.