

Logarithms: The Inverse of Exponents

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: F-LE.A.4, F-BF.B.4a, F-IF.C.7e

Each solution below shows the key step, usually the conversion to an exponent, so that mistakes are easy to trace. Compare every step with the work, not only the final answer, and redo any problem whose method differs.

Standards: F-LE.A.4, F-BF.B.4a, F-IF.C.7e · **Level:** Grade 11 Algebra 2

Problem 1 Exponential to logarithmic form

The base 2 stays the base and the exponent 9 becomes the value of the log. **$\log_2 512 = 9$.**

Problem 2 Logarithmic to exponential form

The base is 6 and the log value 3 is the exponent. **$6^3 = 216$.**

Problem 3 Evaluate a basic logarithm

$3^5 = 243$, so **$\log_3 243 = 5$.**

Problem 4 Logarithm of a power of 10

$0.001 = 10^{-3}$, so **$\log 0.001 = -3$.**

Problem 5 Find the base from a graph

The point (4, 1) means $\log_b 4 = 1$, so $b^1 = 4$ and $b = 4$. The other points agree: $4^2 = 16$ and $4^{-1} = 0.25$. Since $4^3 = 64$, **$b = 4$ and $\log_4 64 = 3$.**

Problem 6 Logarithm with a fractional base

Let $\log_{1/2} 8 = y$. Then $(1/2)^y = 8$. Since $8 = 2^3 = (1/2)^{-3}$, **$y = -3$.**

Problem 7 Logarithm giving a fraction

$25^y = 5$ means $(5^2)^y = 5^1$, so $2y = 1$. **$y = \frac{1}{2}$.**

Problem 8 Natural logarithm of a power of e

ln is the log base e, so $\ln e^4 = 4$. Also $\frac{1}{e} = e^{-1}$, so $\ln \frac{1}{e} = -1$. **4 and -1.**

Problem 9 Expand a logarithm

$\log_3 9 + \log_3 x - \log_3 y^2 = 2 + \log_3 x - 2\log_3 y$. **$2 + \log_3 x - 2 \log_3 y$.**

Problem 10 Condense a logarithmic expression

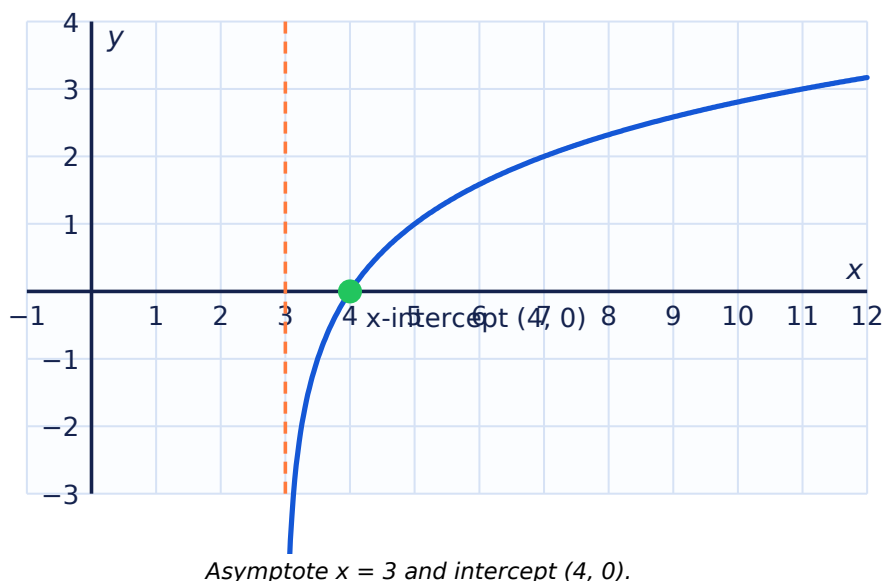
Power rule: $\log_5 m^2 + \log_5 4 - \log_5 n$. Product and quotient: $\log_5(4m^2/n)$.

Problem 11 Change of base

$\log_3 50 = \ln 50 \div \ln 3 \approx 3.9120 \div 1.0986 \approx 3.5609 \approx \mathbf{3.56}$.

Problem 12 Domain and asymptote

The argument must be positive: $x - 3 > 0$, so $x > 3$. The asymptote is where the argument is 0. For the intercept, $\log_2(x - 3) = 0$ gives $x - 3 = 1$. **Domain $x > 3$; asymptote $x = 3$; x-intercept (4, 0).**

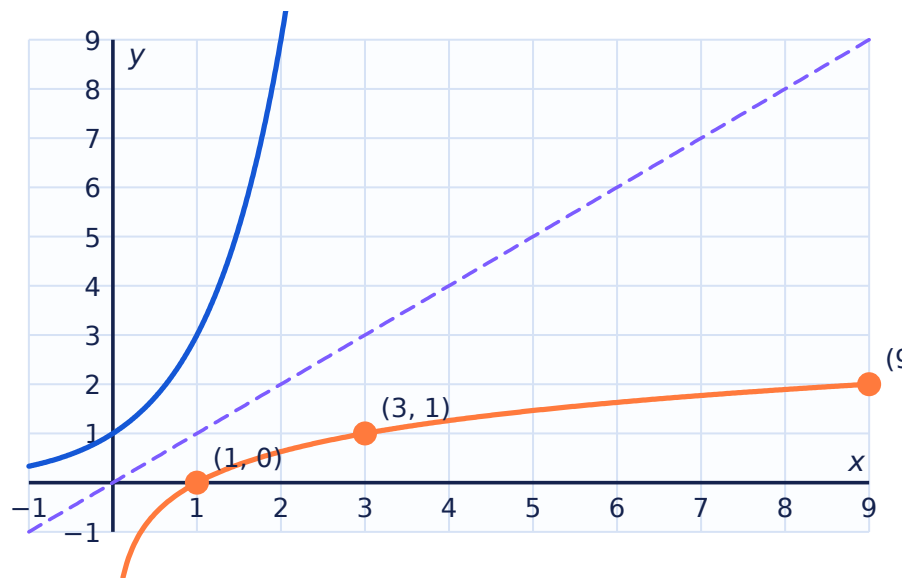


Problem 13 Locate a log on a number line

$1000 = 10^3$ and $10,000 = 10^4$, and $1000 < 5000 < 10,000$. So the log lies between 3 and 4. A calculator gives $\log 5000 \approx 3.699$. **Between 3 and 4; ≈ 3.7 .**

Problem 14 Reflect points to find an inverse

Reflecting over $y = x$ swaps the coordinates: (1, 0), (3, 1) and (9, 2). These satisfy $y = \log_3 x$. **(1, 0), (3, 1), (9, 2); inverse $y = \log_3 x$.**



The inverse points sit on the orange curve.

Problem 15 Earthquake energy comparison

A magnitude difference of $7.1 - 5.9 = 1.2$ means the amplitudes differ by a factor of $10^{1.2}$. $10^{1.2} \approx 15.85$. **About 15.85 times larger.**

Problem 16 Decibel level of a vacuum cleaner

$I/I_0 = 3.2 \times 10^{-5} \div 10^{-12} = 3.2 \times 10^7$. Then $\log(3.2 \times 10^7) = \log 3.2 + 7 \approx 0.5051 + 7 = 7.5051$. $L = 75.051$. **≈ 75.1 dB.**

Problem 17 pH of a soil sample

$\text{pH} = -\log(2.5 \times 10^{-6}) = -(\log 2.5 - 6) = 6 - 0.3979 = 5.6021$. **pH ≈ 5.60 , which is acidic** (below 7).

Problem 18 Inverse of a logarithmic function

Write $y = 2 \log_5 x + 1$, swap: $x = 2 \log_5 y + 1$. Then $\log_5 y = (x - 1)/2$, so $y = 5^{(x-1)/2}$. Check: $f(25) = 2 \cdot 2 + 1 = 5$ and $f^{-1}(5) = 5^2 = 25$. **$f^{-1}(x) = 5^{(x-1)/2}$.**

Problem 19 Using given log values

$18 = 2 \cdot 3^2$, so $\log_b 18 = 0.43 + 2(0.68) = 1.79$. Also $8/9 = 2^3/3^2$, so $\log_b(8/9) = 3(0.43) - 2(0.68) = 1.29 - 1.36 = -0.07$. **1.79 and -0.07.**

Problem 20 Justify a change-of-base consequence

$\log_8 32 = \frac{(\log_2 32)}{(\log_2 8)}$ by change of base. Since $32 = 2^5$ and $8 = 2^3$, the quotient is $5/3$. Check: $8^{5/3} = (2^3)^{5/3} = 2^5 = 32$. **$\log_8 32 = 5/3$.**