

Logarithms: The Inverse of Exponents

Grade 11 – Algebra 2 math · Worksheet · Standards: F-LE.A.4, F-BF.B.4a, F-IF.C.7e

Name: _____ Date: _____ Class: _____

These problems train the link between exponents and logarithms, the properties of logarithms, and reading graphs of logarithmic functions, as in Common Core standards F-LE.A.4, F-BF.B.4a and F-IF.C.7e.

They move from converting forms to modeling with pH, decibels and earthquake magnitude. Work the first ten without a calculator wherever possible and keep every step of the algebra.

Standards: F-LE.A.4, F-BF.B.4a, F-IF.C.7e · **Level:** Grade 11 Algebra 2

Problem 1 Exponential to logarithmic form (easy)

Write $2^9 = 512$ in logarithmic form.

Problem 2 Logarithmic to exponential form (easy)

Write $\log_6 216 = 3$ in exponential form.

Problem 3 Evaluate a basic logarithm (easy)

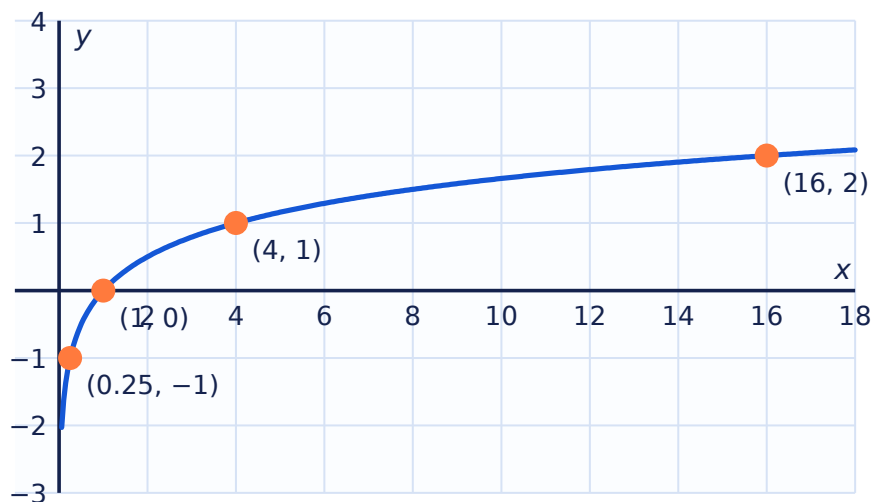
Evaluate $\log_3 243$.

Problem 4 Logarithm of a power of 10 (easy)

Evaluate $\log 0.001$ without a calculator.

Problem 5 Find the base from a graph (medium)

The graph below shows $y = \log_b x$. Use the point $(4, 1)$ to find b , then give the value of $\log_b 64$.



The graph of $y = \log_b x$ for an unknown base b .

Problem 6 Logarithm with a fractional base (medium)

Evaluate $\log_{1/2} 8$.

Problem 7 Logarithm giving a fraction (medium)

Evaluate $\log_{25} 5$.

Problem 8 Natural logarithm of a power of e (medium)

Evaluate $\ln e^4$ and $\ln \frac{1}{e}$.

Problem 9 Expand a logarithm (medium)

Expand $\log_3\left(\frac{9x}{y^2}\right)$ so that no product, quotient or power remains inside a log.

Problem 10 Condense a logarithmic expression (medium)

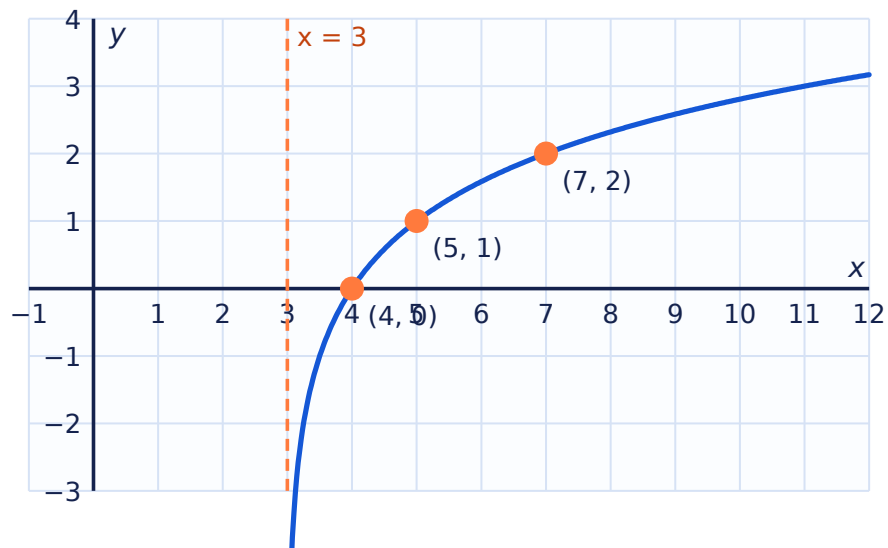
Write $2 \log_5 m + \log_5 4 - \log_5 n$ as a single logarithm.

Problem 11 Change of base (medium)

Use the change-of-base formula to approximate $\log_3 50$ to the nearest hundredth.

Problem 12 Domain and asymptote (medium)

The graph below is $y = \log_2(x - 3)$. State the domain, the equation of the vertical asymptote and the x-intercept.



A shifted logarithmic graph with a dashed vertical asymptote.

Problem 13 Locate a log on a number line (medium)

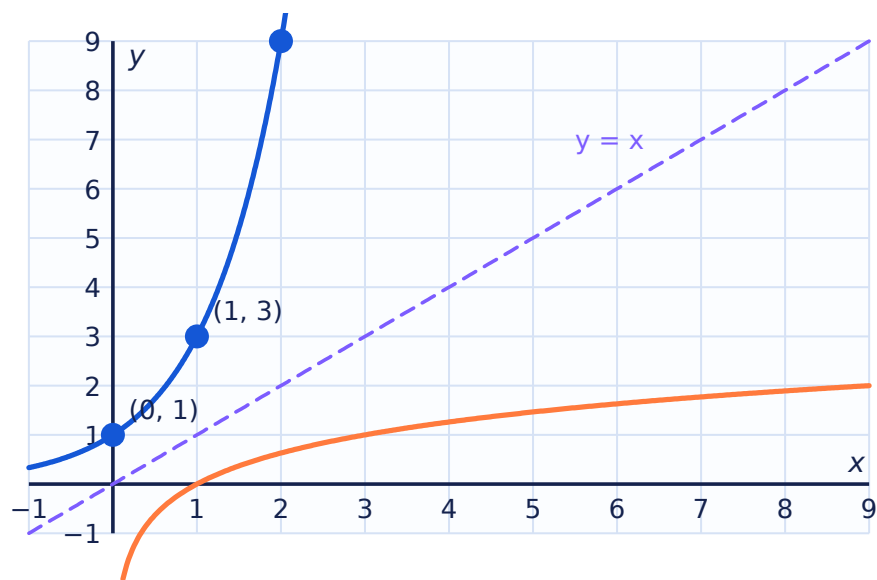
On the number line, $\log 5000$ lies between two consecutive integers. Name them, then use a calculator to give $\log 5000$ to the nearest tenth.



A number line with $\log 5000$ marked.

Problem 14 Reflect points to find an inverse (hard)

The graph of $y = 3^x$ contains $(0, 1)$, $(1, 3)$ and $(2, 9)$. List the corresponding points of the inverse and write the inverse function.



The curve $y = 3^x$, the line $y = x$ and the inverse curve.

Problem 15 Earthquake energy comparison (hard)

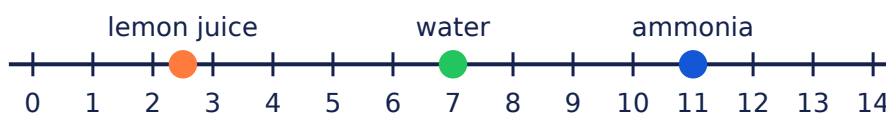
Earthquake magnitude M is a base-10 log of shaking amplitude. A quake of magnitude 7.1 is compared with one of magnitude 5.9. How many times larger is the amplitude of the stronger quake? Round to the nearest hundredth.

Problem 16 Decibel level of a vacuum cleaner (hard)

Sound level in decibels is $L = 10 \log(I/I_0)$, where $I_0 = 10^{-12} \text{ W/m}^2$. A vacuum cleaner produces an intensity $I = 3.2 \times 10^{-5} \text{ W/m}^2$. Find its sound level to the nearest tenth.

Problem 17 pH of a soil sample (hard)

The pH of a liquid is $-\log[H^+]$, where $[H^+]$ is in moles per liter. A rainwater sample has $[H^+] = 2.5 \times 10^{-6}$. Find its pH to the nearest hundredth. Is it acidic (pH below 7)?



The pH scale for comparison.

Problem 18 Inverse of a logarithmic function (hard)

Let $f(x) = 2 \log_5 x + 1$. Find $f^{-1}(x)$ and verify it by computing $f^{-1}(f(25))$.

Problem 19 Using given log values (hard)

Suppose $\log_b 2 = 0.43$ and $\log_b 3 = 0.68$. Find $\log_b 18$ and $\log_b(\frac{8}{9})$ using the properties of logarithms.

Problem 20 Justify a change-of-base consequence (hard)

Show that $\log_8 32 = \frac{5}{3}$ by rewriting both numbers as powers of 2 and using the change-of-base idea.