

Normal Curves, z-Scores and Study Design

Grade 11 – Algebra 2 math · Lesson · Standards: S-ID.A.4, S-IC.A.1, S-IC.B.3, S-IC.B.4

Quick answer: Standard deviation measures how far data typically fall from the mean. In a normal distribution, about 68% of values lie within 1 standard deviation of the mean, 95% within 2, and 99.7% within 3. A z-score, $z = (x - \mu)/\sigma$, tells how many standard deviations a value is from the mean, so values on different scales can be compared.

Standards: S-ID.A.4, S-IC.A.1, S-IC.B.3, S-IC.B.4 · **Level:** Grade 11 Algebra 2

What do the mean and standard deviation tell you?

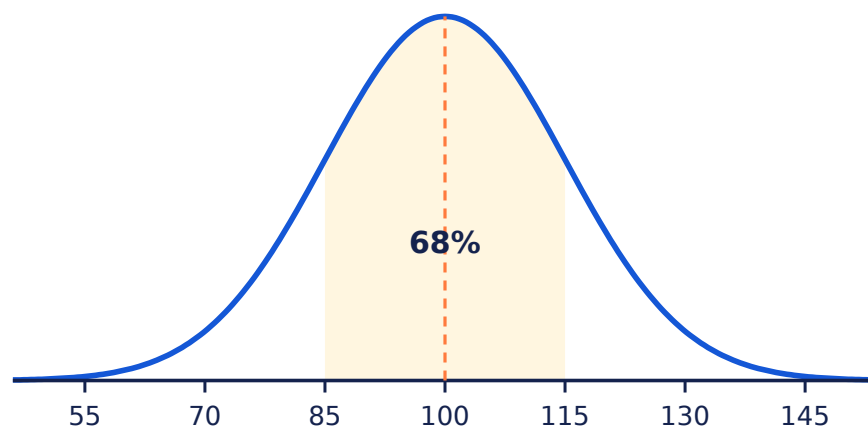
The mean μ is the balance point of the data. The standard deviation σ measures the typical distance from the mean. To compute it, square each deviation, average the squares, then take the square root.

Example

Data: 6, 8, 10, 12, 14. Mean = 10. Deviations: $-4, -2, 0, 2, 4$, with squares 16, 4, 0, 4, 16. Average = $40 \div 5 = 8$, so $\sigma = \sqrt{8} \approx \mathbf{2.83}$.

What is the 68-95-99.7 rule?

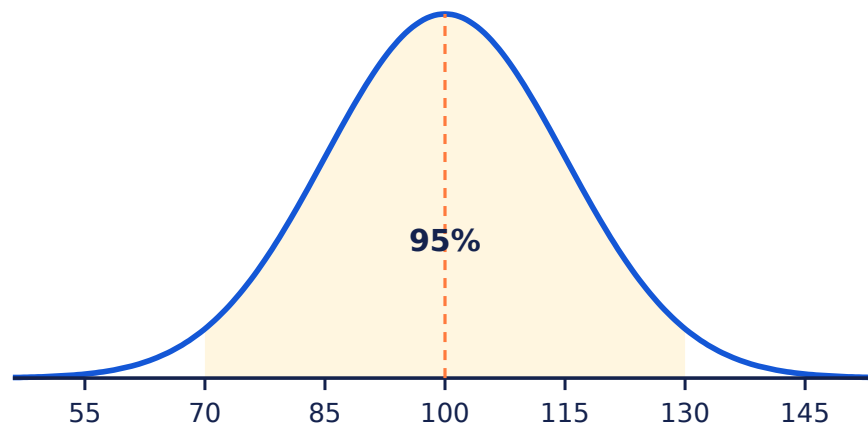
For a normal distribution (a symmetric bell curve), almost all values sit near the mean.



About 68% of values lie within one standard deviation of the mean.

Empirical Rule

- About 68% of values lie within 1σ of the mean.
- About 95% lie within 2σ .
- About 99.7% lie within 3σ .



About 95% lie within two standard deviations.

How do z-scores compare values?

z-score

$$z = (x - \mu) \div \sigma$$

It counts how many standard deviations a value is above (+) or below (–) the mean.

Method

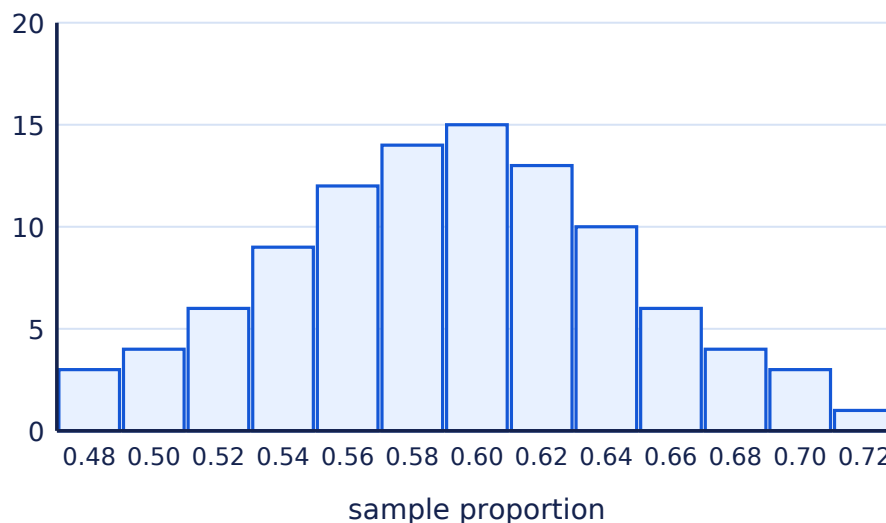
1. Compute the z-score of each value with its own mean and σ .
2. Compare the z-scores, not the raw scores.
3. Flag $|z| > 2$ as unusual.

How do surveys, experiments and observational studies differ?

A survey samples a population to estimate a parameter, and random selection reduces bias. An observational study records what happens without assigning anything. An experiment randomly assigns treatments, which is the only design that can support a cause-and-effect claim.

What is a margin of error?

Repeating a random sample many times by simulation shows how much a sample proportion varies. The middle 95% of the simulated results spans about ± 2 standard deviations, and half that width is the margin of error.



Results of 100 simulated samples of 50 people when the true proportion is 0.60.

Common mistake

Comparing raw scores from tests with different means and spreads. A higher score is not always the better performance.

Mathbot's tip

Draw the bell curve and mark $\mu \pm \sigma$, 2σ and 3σ . The picture shows the percent you need.

Key points to remember

- σ is the square root of the average squared deviation from the mean.
- Empirical Rule: 68%, 95% and 99.7% within 1, 2 and 3 standard deviations.
- $z = (x - \mu)/\sigma$; $|z|$ greater than 2 marks an unusual value.
- Only a randomized experiment can support a cause-and-effect conclusion.
- Simulating many random samples shows the margin of error: about twice the standard deviation of the sample proportions.

Normal Curves, z-Scores and Study Design: frequently asked questions

What does a z-score of -1.5 mean?

The value lies 1.5 standard deviations below the mean. In a normal distribution it is somewhat below average but not unusual, because only values beyond $z = \pm 2$ are rare.

Why divide by σ in a z-score?

Dividing by σ removes the units and the spread of the data. A z-score of 2 means the same position, two standard deviations above the mean, whether the data are test scores or heights.

When can I use the 68-95-99.7 rule?

Use it when the data distribution is roughly symmetric and bell-shaped. For skewed data the percentages can be far off, so check a histogram first.

What is the difference between a survey and an experiment?

A survey asks questions of a sample to learn about a population. An experiment assigns different treatments to subjects at random and compares results, which lets it show cause and effect.

How does simulation give a margin of error?

Simulate many random samples of the same size, record each sample proportion, and look at the spread. The interval containing the middle 95% of results has half-width equal to the margin of error.