

Normal Curves, z-Scores and Study Design

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: S-ID.A.4, S-IC.A.1, S-IC.B.3, S-IC.B.4

Each solution states the z-score or the standard-deviation distances before using the Empirical Rule. For simulation problems, compare your interval with the shaded bars and accept answers within one bar.

Standards: S-ID.A.4, S-IC.A.1, S-IC.B.3, S-IC.B.4 · **Level:** Grade 11 Algebra 2

Problem 1 Mean of a data set

$(23 + 31 + 28 + 19 + 34) \div 5 = 135 \div 5$. **27 loaves.**

Problem 2 Standard deviation of a small set

Mean 9. Squared deviations: 36, 16, 0, 16, 36, which sum to 104. Variance = $104/5 = 20.8$ and $\sigma = \sqrt{20.8} = 4.561$. **≈ 4.56 .**

Problem 3 Compute a z-score

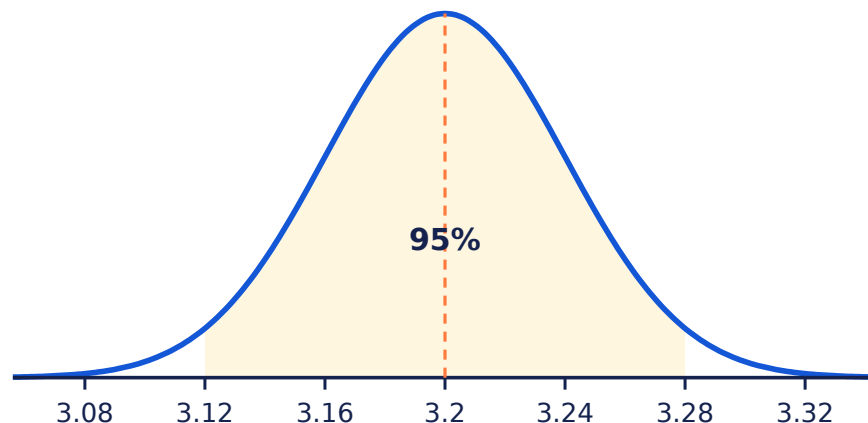
$z = (84 - 72)/6 = 2$. **$z = 2$.**

Problem 4 Value from a z-score

$x = 500 + (-1.5)(80) = 500 - 120$. **380.**

Problem 5 Applying the 68-95 rule

68%: 3.2 ± 0.04 gives 3.16 to 3.24. 95%: 3.2 ± 0.08 gives 3.12 to 3.28. **3.16-3.24 in and 3.12-3.28 in.**



The 95% region runs from 3.12 to 3.28.

Problem 6 Percent beyond one standard deviation

36 is one standard deviation above the mean. 68% are within 1σ , so 32% are outside, split evenly: 16% above. **About 16%.**

Problem 7 Between two values

$60 = 76 - 2(8)$ and $92 = 76 + 2(8)$. **About 95%.**

Problem 8 Counting students

58 and 78 are 1σ below and above 68. That is 68% of the students: $0.68 \times 400 = 272$. **About 272 students.**

Problem 9 Compare with z-scores

Mia: $z = (88 - 80)/4 = 2$. Noah: $z = (91 - 82)/6 = 1.5$. **Mia performed better relative to her class.**

Problem 10 Identify the study type

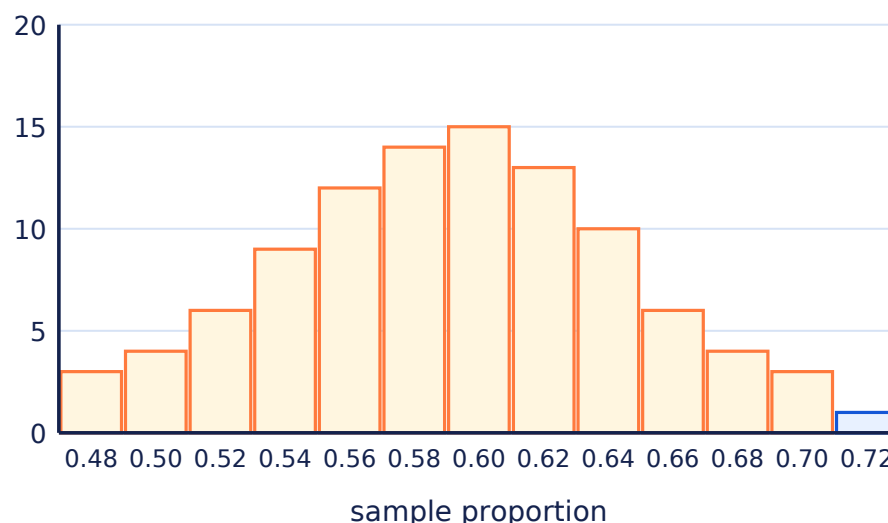
(a) Sampling a population to ask opinions is a survey. (b) The researcher only records, with no assigned treatment: observational. (c) Random assignment of a treatment: experiment. **(a) survey, (b) observational, (c) experiment.**

Problem 11 Spot the bias

Students at the gym are not a random sample of the school, so athletes are over-represented and may differ in food preferences. A better method is a simple random sample, such as picking 30 student ID numbers at random. **Convenience sample; use a random sample.**

Problem 12 Margin of error from a simulation

Cumulative counts show about 2 or 3 results in each tail. The middle 95% lies roughly from 0.48 to 0.70. The margin of error is half the width: $(0.70 - 0.48)/2 = 0.11$. **About 0.48 to 0.70; margin of error ≈ 0.11 .**



Problem 13 Mean and deviation from a frequency table

Mean = $(6 + 20 + 24)/10 = 5$. Squared deviations: $3(9) + 4(0) + 3(9) = 54$, variance 5.4, $\sigma = \sqrt{5.4} = 2.3238$. **Mean 5; $\sigma \approx 2.32$.**

Problem 14 Find the mean from a z-score

$93 = \mu + 1.75(8) = \mu + 14$, so $\mu = 79$. **Mean 79.**

Problem 15 Percentiles from the rule

95% lie within 2σ , so 5% lie outside, 2.5% in each tail. Upper: $55 + 2(2.4) = 59.8$. Lower: $55 - 4.8 = 50.2$. **59.8 in and 50.2 in.**

Problem 16 Effect of an outlier

Before: mean 15.4, $\sigma = \sqrt{1.04} \approx 1.02$. After: mean 19.5, $\sigma = \sqrt{(84.917)} \approx 9.22$. **The mean rises from 15.4 to 19.5 and σ rises from 1.02 to 9.22:** one outlier greatly inflates both.

Problem 17 Design an experiment

Randomly assign 30 students to use the app (treatment) and 30 to study as usual (control), for example by drawing names. Give both groups the same quiz and compare the mean scores. Random assignment balances other factors, so a difference can be attributed to the app.
Random assignment to treatment and control groups; compare mean quiz scores.

Problem 18 Margin of error by formula

$\sqrt{(0.55 \times 0.45/40)} = \sqrt{0.0061875} = 0.0787$. Margin $\approx 2 \times$ that = 0.157. Interval: 0.55 ± 0.16 , or 0.39 to 0.71. **Margin ≈ 0.16 ; interval ≈ 0.39 to 0.71 .**

Problem 19 Unusual temperatures

97: $z = (97 - 86)/5 = 2.2$, beyond 2, so unusual. 78: $z = (78 - 86)/5 = -1.6$, within 2, so not unusual. **$z = 2.2$ (unusual) and $z = -1.6$ (not unusual).**

Problem 20 Compare two classes

Class A: $z = 15/5 = 3$. Class B: $z = 15/12 = 1.25$. In Class A, 85 is 3σ above the mean, which almost never happens (about 0.15%). In Class B it is only 1.25σ . **Class B is likelier to have students above 85.**