

Dividing Polynomials: Remainder and Factor Theorems

Grade 11 – Algebra 2 math · Test Answer Key · Standards: A-APR.B.2, A-APR.D.6, A-APR.C.4

Each solution shows the division or substitution and the scoring guide. Accept any correct method, such as the Remainder Theorem in place of division when only a remainder is requested, and award partial credit for correct setup.

Standards: A-APR.B.2, A-APR.D.6, A-APR.C.4 · **Level:** Grade 11 Algebra 2

Question 1 Long division (3 points)

Divide x^3 by x to get x^2 . Multiply: $x^2(x + 3) = x^3 + 3x^2$. Subtract: $-5x^2 - 9x + 25$.

Divide $-5x^2$ by x to get $-5x$. Multiply: $-5x(x + 3) = -5x^2 - 15x$. Subtract: $6x + 25$.

Divide $6x$ by x to get 6 . Multiply: $6(x + 3) = 6x + 18$. Subtract: 7 .

Quotient $x^2 - 5x + 6$, remainder 7

Scoring: 1 point for each correct division step, up to 2, and 1 point for the quotient and remainder.

Question 2 Synthetic division (3 points)

The x^3 term is missing, so use the coefficients 3, 0, -2 , 5, -9 .

Use $r = -2$. Bring down 3. Then multiply by -2 and add down each column.

Bottom row: 3, -6 , 10, -15 , 21.

Quotient $3x^3 - 6x^2 + 10x - 15$, remainder 21

Scoring: 1 point for including the zero coefficient, 1 for the table, 1 for the quotient and remainder.

Question 3 Remainder Theorem (2 points)

$$f(2) = 16 - 24 + 4 - 5 = -9.$$

$$f(-1) = 1 + 3 - 2 - 5 = -3.$$

Remainders –9 and –3

Scoring: 1 point each remainder.

Question 4 Factor Theorem (3 points)

$f(4) = 64 - 80 - 8 + 24 = 0$, so $x - 4$ is a factor.

Use $r = 4$. Bring down 1. Then multiply by 4 and add down each column.

Bottom row: 1, –1, –6, 0.

Quotient $x^2 - x - 6$, remainder 0

$$x^2 - x - 6 = (x - 3)(x + 2).$$

$$f(x) = (x - 4)(x - 3)(x + 2)$$

Scoring: 1 point for $f(4) = 0$, 1 for the quotient, 1 for complete factoring.

Question 5 An unknown coefficient (2 points)

The value at $x = -3$ must be 0: $-27 + 9k + 6 + 12 = 0$, so $9k - 9 = 0$.

$$k = 1$$

Scoring: 1 point for the equation, 1 point for k .

Question 6 Polynomial identity (3 points)

a) Left: $x^4 + 2x^2y^2 + y^4$. Right: $x^4 - 2x^2y^2 + y^4 + 4x^2y^2 = x^4 + 2x^2y^2 + y^4$. Equal.

b) Legs $16 - 1 = 15$ and $2(4)(1) = 8$; hypotenuse $16 + 1 = 17$. Check: $225 + 64 = 289$.

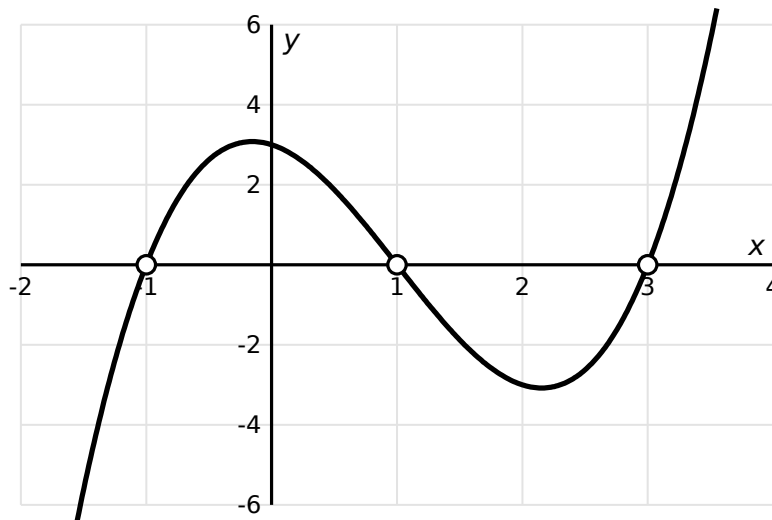
Sides 15, 8, 17

Scoring: 1 point for each expansion in (a), 1 point for (b).

Question 7 Remainders from a graph (2 points)

a) By the Remainder Theorem the remainders are $f(2) = -3$ and $f(3) = 0$.

b) The zeros –1, 1, and 3 give the factors $x + 1$, $x - 1$, $x - 3$.



Zeros of f .

Scoring: 1 point each part.

Question 8 Volume of a prism (2 points)

Divide x^3 by x to get x^2 . Multiply: $x^2(x + 2) = x^3 + 2x^2$. Subtract: $7x^2 + 26x + 24$.

Divide $7x^2$ by x to get $7x$. Multiply: $7x(x + 2) = 7x^2 + 14x$. Subtract: $12x + 24$.

Divide $12x$ by x to get 12 . Multiply: $12(x + 2) = 12x + 24$. Subtract: 0 .

Quotient $x^2 + 7x + 12$, remainder 0

$$x^2 + 7x + 12 = (x + 3)(x + 4).$$

Base area $x^2 + 7x + 12$; length $x + 4$, width $x + 3$

Scoring: 1 point for the division, 1 point for the factoring.