

Dividing Polynomials: Remainder and Factor Theorems

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: A-APR.B.2, A-APR.D.6, A-APR.C.4

Each solution shows the division steps or substitution in full. Multiply the divisor by the quotient and add the remainder to confirm the original polynomial.

Standards: A-APR.B.2, A-APR.D.6, A-APR.C.4 · **Level:** Grade 11 Algebra 2

Problem 1 Long division with no remainder

Divide x^2 by x to get x . Multiply: $x(x + 4) = x^2 + 4x$. Subtract: $7x + 28$.

Divide $7x$ by x to get 7 . Multiply: $7(x + 4) = 7x + 28$. Subtract: 0 .

Quotient $x + 7$, remainder 0

Problem 2 Long division with a leading coefficient

Divide $2x^2$ by x to get $2x$. Multiply: $2x(x + 4) = 2x^2 + 8x$. Subtract: $-3x - 12$.

Divide $-3x$ by x to get -3 . Multiply: $-3(x + 4) = -3x - 12$. Subtract: 0 .

Quotient $2x - 3$, remainder 0

Problem 3 A remainder appears

Divide x^2 by x to get x . Multiply: $x(x - 2) = x^2 - 2x$. Subtract: $8x - 5$.

Divide $8x$ by x to get 8 . Multiply: $8(x - 2) = 8x - 16$. Subtract: 11 .

Quotient $x + 8$, remainder 11

So $(x^2 + 6x - 5) \div (x - 2) = x + 8 + \frac{11}{(x - 2)}$.

Problem 4 Completing a synthetic division

Products: $2 \cdot 2 = 4$, $2 \cdot (-1) = -2$, $2 \cdot (-3) = -6$. Sums: $-5 + 4 = -1$, $-1 + (-2) = -3$, $9 + (-6) =$

3.

2	2	-5	-1	9
		4	-2	-6
	2	-1	-3	3

remainder

The completed table.

Quotient $2x^2 - x - 3$, remainder 3

Problem 5 Synthetic division with a zero remainder

Use $r = 2$. Bring down 1. Then multiply by 2 and add down each column.

Bottom row: 1, 4, -5, 0.

Quotient $x^2 + 4x - 5$, remainder 0

Problem 6 Synthetic division by x plus a number

Use $r = -2$. Bring down 3. Then multiply by -2 and add down each column.

Bottom row: 3, -7, 18, -43.

Quotient $3x^2 - 7x + 18$, remainder -43

Problem 7 A missing term

The x^3 term is missing, so use the coefficients 1, 0, -5, 2, 8.

Use $r = 3$. Bring down 1. Then multiply by 3 and add down each column.

Bottom row: 1, 3, 4, 14, 50.

Quotient $x^3 + 3x^2 + 4x + 14$, remainder 50

Problem 8 Dividing by $2x + 1$

Divide $6x^3$ by $2x$ to get $3x^2$. Multiply: $3x^2(2x + 1) = 6x^3 + 3x^2$. Subtract: $-2x^2 + 7x + 9$.

Divide $-2x^2$ by $2x$ to get $-x$. Multiply: $-x(2x + 1) = -2x^2 - x$. Subtract: $8x + 9$.

Divide $8x$ by $2x$ to get 4. Multiply: $4(2x + 1) = 8x + 4$. Subtract: 5.

Quotient $3x^2 - x + 4$, remainder 5

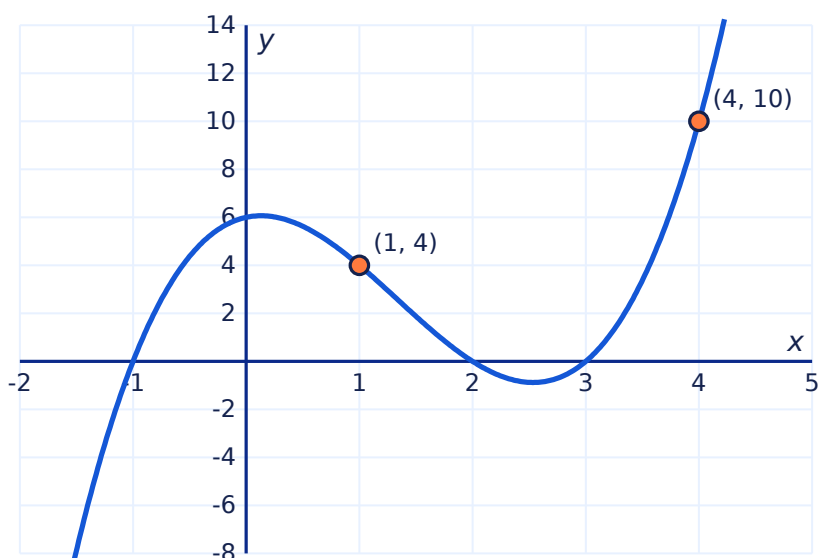
Problem 9 Reading a remainder from a graph

The graph passes through (4, 10) and (1, 4). By the Remainder Theorem, $f(4) = 10$ and $f(1) = 4$.

Use $r = 4$. Bring down 1. Then multiply by 4 and add down each column.

Bottom row: 1, 0, 1, 10.

Quotient $x^2 + 1$, remainder 10



Remainders are heights of the graph.

Remainders: 10 for $x - 4$ and 4 for $x - 1$

Problem 10 Evaluating with the Remainder Theorem

$$f(3) = 2(27) - 5(9) + 3 - 8 = 54 - 45 + 3 - 8 = 4.$$

$$f(-2) = 2(-8) - 5(4) - 2 - 8 = -16 - 20 - 2 - 8 = -46.$$

Remainders 4 and -46

Problem 11 Using the Factor Theorem

$$f(-3) = -27 + 9 + 30 + 8 = 20 \neq 0.$$

$$f(1) = 1 + 1 - 10 + 8 = 0.$$

$$f(-4) = -64 + 16 + 40 + 8 = 0.$$

$x - 1$ and $x + 4$ are factors; $x + 3$ is not

Problem 12 Finding an unknown coefficient

No remainder means the value at $x = 2$ is 0: $8 - 4k + 18 - 14 = 0$, so $12 - 4k = 0$ and $k = 3$.

Check: $x^3 - 3x^2 + 9x - 14 = (x - 2)(x^2 - x + 7)$.

$k = 3$

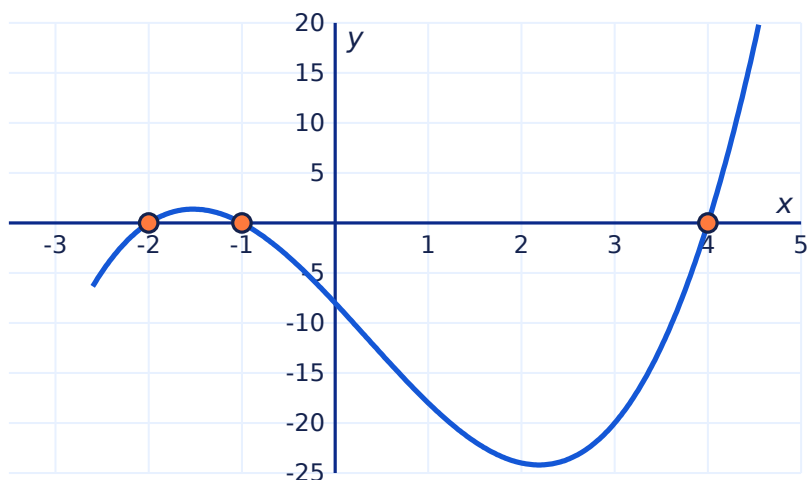
Problem 13 Factoring completely from a zero

Use $r = -2$. Bring down 1. Then multiply by -2 and add down each column.

Bottom row: 1, -3 , -4 , 0.

Quotient $x^2 - 3x - 4$, remainder 0

The quotient $x^2 - 3x - 4 = (x - 4)(x + 1)$.



Zeros at -2 , -1 , and 4 .

$f(x) = (x + 2)(x + 1)(x - 4)$

Problem 14 Dividing by a quadratic

Divide x^4 by x^2 to get x^2 . Multiply: $x^2(x^2 + 2x - 1) = x^4 + 2x^3 - x^2$. Subtract: $x^3 - x^2 - 3x + 1$.

Divide x^3 by x^2 to get x . Multiply: $x(x^2 + 2x - 1) = x^3 + 2x^2 - x$. Subtract: $-3x^2 - 2x + 1$.

Divide $-3x^2$ by x^2 to get -3 . Multiply: $-3(x^2 + 2x - 1) = -3x^2 - 6x + 3$. Subtract: $4x - 2$.

Quotient $x^2 + x - 3$, remainder $4x - 2$

Problem 15 A divisor with leading coefficient 2

Coefficients 6, -7 , 5, -7 with $r = \frac{3}{2}$: bring down 6; $6 \cdot \frac{3}{2} = 9$, $-7 + 9 = 2$; $2 \cdot \frac{3}{2} = 3$, $5 + 3 = 8$; $8 \cdot \frac{3}{2} = 12$, $-7 + 12 = 5$.

So $f(x) = (x - \frac{3}{2})(6x^2 + 2x + 8) + 5 = (2x - 3)(3x^2 + x + 4) + 5$.

Quotient $3x^2 + x + 4$, remainder 5

Problem 16 Dimensions of a box

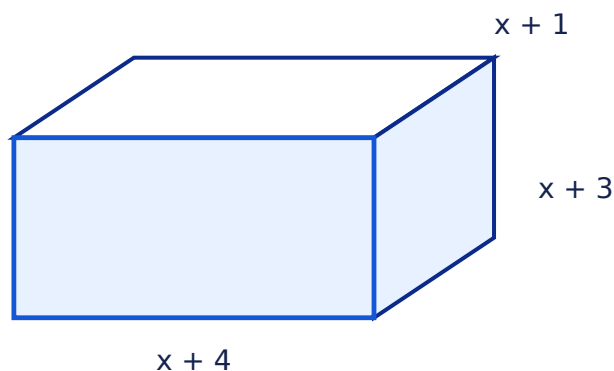
Divide x^3 by x to get x^2 . Multiply: $x^2(x + 3) = x^3 + 3x^2$. Subtract: $5x^2 + 19x + 12$.

Divide $5x^2$ by x to get $5x$. Multiply: $5x(x + 3) = 5x^2 + 15x$. Subtract: $4x + 12$.

Divide $4x$ by x to get 4. Multiply: $4(x + 3) = 4x + 12$. Subtract: 0.

Quotient $x^2 + 5x + 4$, remainder 0

Base area = $x^2 + 5x + 4 = (x + 1)(x + 4)$.



Dimensions $x + 4$, $x + 1$, $x + 3$.

Base area $x^2 + 5x + 4$; length $x + 4$ and width $x + 1$

Problem 17 Break-even points

Use $r = 1$. Bring down 1. Then multiply by 1 and add down each column.

Bottom row: 1, -6 , 8, 0.

Quotient $x^2 - 6x + 8$, remainder 0

The quotient $x^2 - 6x + 8 = (x - 2)(x - 4)$.

Break-even at 100, 200, and 400 meals ($x = 1, 2, 4$)

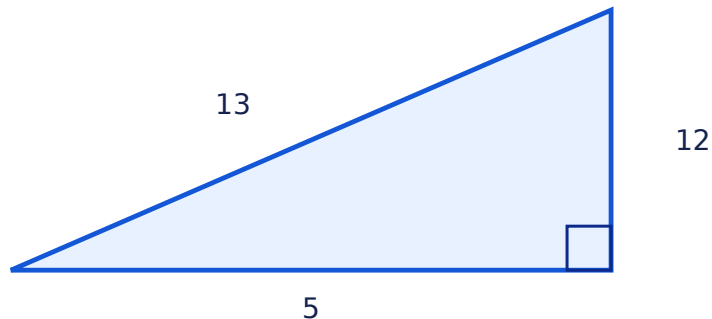
Problem 18 Proving an identity

Left side: $x^4 + 2x^2y^2 + y^4$.

Right side: $(x^4 - 2x^2y^2 + y^4) + 4x^2y^2 = x^4 + 2x^2y^2 + y^4$.

The sides match, so the identity holds.

With $x = 3$, $y = 2$: $x^2 - y^2 = 5$, $2xy = 12$, $x^2 + y^2 = 13$, and $5^2 + 12^2 = 169 = 13^2$.



The triangle for $x = 3$, $y = 2$.

Triple (5, 12, 13)

Problem 19 Using identities for calculation

a) $x^2 - y^2 = 21$, $2xy = 20$, $x^2 + y^2 = 29$. Check: $441 + 400 = 841 = 29^2$. **Triple (20, 21, 29)**.

b) $(1,003 - 997)(1,003 + 997) = 6 \cdot 2,000 = \mathbf{12,000}$.

Problem 20 Finding two unknown coefficients

$p(1) = 0$: $1 + a + b + 6 = 0$, so $a + b = -7$.

$p(-2) = 0$: $-8 + 4a - 2b + 6 = 0$, so $2a - b = 1$.

Add the two equations: $3a = -6$, $a = -2$, and then $b = -5$.

Then $p(x) = x^3 - 2x^2 - 5x + 6$, and dividing by $(x - 1)(x + 2) = x^2 + x - 2$ leaves $x - 3$.

$a = -2$, $b = -5$, third factor $x - 3$