

# Dividing Polynomials: Remainder and Factor Theorems

Grade 11 – Algebra 2 math · Worksheet · Standards: A-APR.B.2, A-APR.D.6, A-APR.C.4

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

These problems cover long division and synthetic division of polynomials, the Remainder and Factor Theorems, and polynomial identities. They address Common Core standards A-APR.B.2, A-APR.D.6, and A-APR.C.4.

Early problems divide step by step; later ones use missing terms, non-monic divisors, graphs, and modeling contexts. Check every quotient by multiplying it by the divisor and adding the remainder.

**Standards:** A-APR.B.2, A-APR.D.6, A-APR.C.4 · **Level:** Grade 11 Algebra 2

## Problem 1 Long division with no remainder (easy)

Use long division to divide  $x^2 + 11x + 28$  by  $x + 4$ .

## Problem 2 Long division with a leading coefficient (easy)

Divide  $2x^2 + 5x - 12$  by  $x + 4$  using long division.

## Problem 3 A remainder appears (easy)

Divide  $x^2 + 6x - 5$  by  $x - 2$  and write the result as quotient plus remainder over divisor.

## Problem 4 Completing a synthetic division (easy)

The table shows the start of dividing  $2x^3 - 5x^2 - x + 9$  by  $x - 2$ . Complete the table, then write the quotient and the remainder.

|   |   |    |    |   |
|---|---|----|----|---|
| 2 | 2 | -5 | -1 | 9 |
|   |   | ?  | ?  | ? |
|   | 2 | ?  | ?  | ? |

remainder

Fill in each ?.

### Problem 5 Synthetic division with a zero remainder (easy)

Use synthetic division to divide  $x^3 + 2x^2 - 13x + 10$  by  $x - 2$ .

### Problem 6 Synthetic division by x plus a number (medium)

Divide  $3x^3 - x^2 + 4x - 7$  by  $x + 2$  using synthetic division. Rewrite the divisor as  $x - (-2)$  first.

### Problem 7 A missing term (medium)

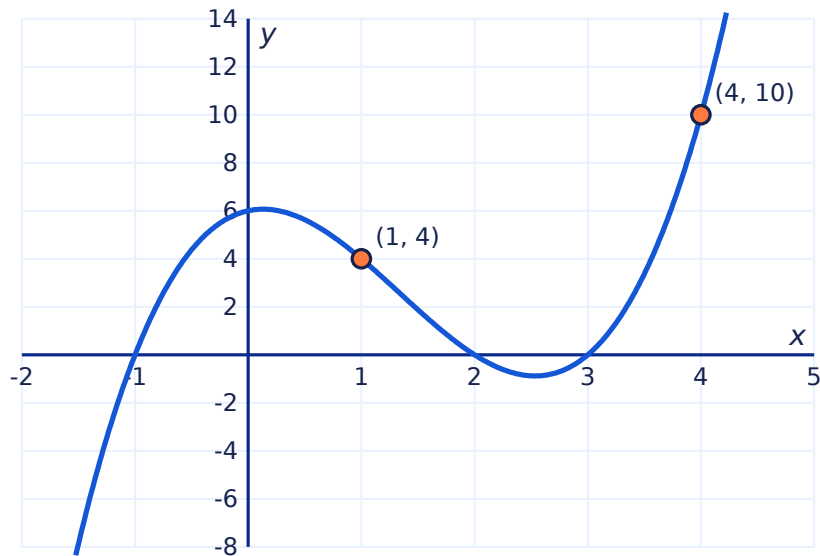
Divide  $x^4 - 5x^2 + 2x + 8$  by  $x - 3$  using synthetic division. Write the quotient and remainder.

### Problem 8 Dividing by $2x + 1$ (medium)

Use long division to divide  $6x^3 + x^2 + 7x + 9$  by  $2x + 1$ .

### Problem 9 Reading a remainder from a graph (medium)

The graph of  $f(x) = x^3 - 4x^2 + x + 6$  is shown. Use the Remainder Theorem and the graph to state the remainder when  $f(x)$  is divided by  $x - 4$  and by  $x - 1$ . Then confirm the first remainder with synthetic division.



Graph of  $f$ .

### Problem 10 Evaluating with the Remainder Theorem (medium)

For  $f(x) = 2x^3 - 5x^2 + x - 8$ , find the remainder when  $f$  is divided by  $x - 3$  and by  $x + 2$ , without dividing.

### Problem 11 Using the Factor Theorem (medium)

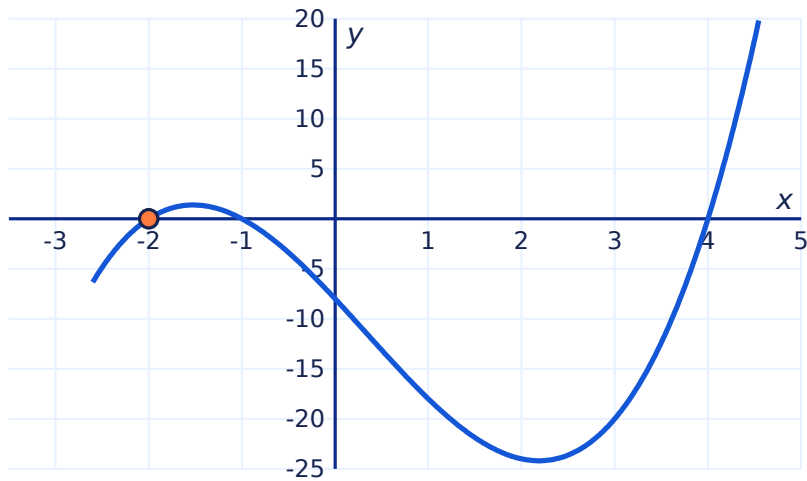
Which of  $x + 3$ ,  $x - 1$ , and  $x + 4$  are factors of  $x^3 + x^2 - 10x + 8$ ? Show the evaluations.

### Problem 12 Finding an unknown coefficient (medium)

For what value of  $k$  does  $x - 2$  divide  $x^3 - kx^2 + 9x - 14$  with no remainder?

### Problem 13 Factoring completely from a zero (medium)

The graph of  $f(x) = x^3 - x^2 - 10x - 8$  is shown and  $-2$  is a zero. Use synthetic division to factor  $f$  completely, and confirm the zeros on the graph.



A cubic graph.

#### Problem 14 Dividing by a quadratic (hard)

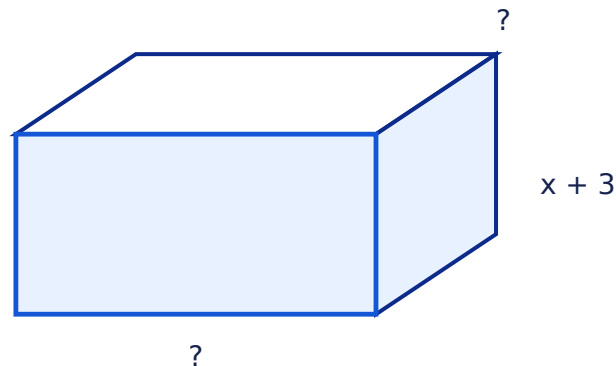
Use long division to divide  $x^4 + 3x^3 - 2x^2 - 3x + 1$  by  $x^2 + 2x - 1$ .

#### Problem 15 A divisor with leading coefficient 2 (hard)

Divide  $6x^3 - 7x^2 + 5x - 7$  by  $2x - 3$  using synthetic division with  $r = \frac{3}{2}$ . Then convert the result to a quotient and remainder for the divisor  $2x - 3$ .

#### Problem 16 Dimensions of a box (hard)

A rectangular box has volume  $V(x) = x^3 + 8x^2 + 19x + 12$  cubic inches and height  $x + 3$  inches. Find an expression for the area of the base, then factor it to find the length and width.



A box with height  $x + 3$ .

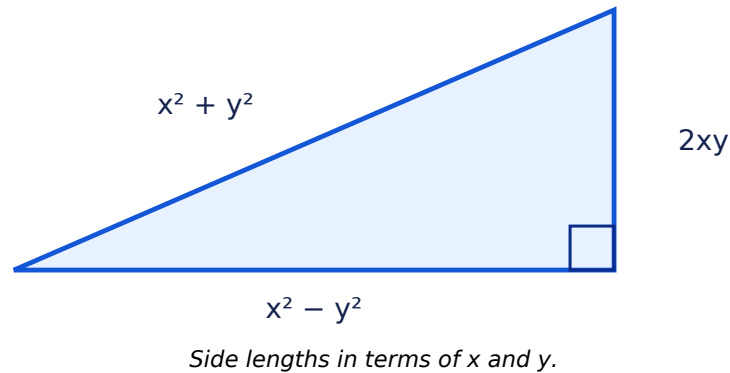
#### Problem 17 Break-even points (hard)

The net profit of a food truck, in thousands of dollars, is modeled by  $P(x) = x^3 - 7x^2 + 14x - 8$ , where  $x$  is the number of hundreds of meals sold. Show that  $x = 1$  gives zero profit, and find all

break-even points.

### Problem 18 Proving an identity (hard)

Prove that  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  by expanding both sides. Then use  $x = 3$  and  $y = 2$  to write a Pythagorean triple and draw the matching triangle labels.



### Problem 19 Using identities for calculation (hard)

- (a) Use  $x = 5$  and  $y = 2$  in the identity from the previous problem to find a Pythagorean triple.
- (b) Use the identity  $a^2 - b^2 = (a - b)(a + b)$  to compute  $1,003^2 - 997^2$  without a calculator.

### Problem 20 Finding two unknown coefficients (hard)

The polynomial  $p(x) = x^3 + ax^2 + bx + 6$  has  $x - 1$  and  $x + 2$  as factors. Find  $a$  and  $b$ , and then find the third factor.