

Polynomial Functions: Zeros, Ends, and Graphs

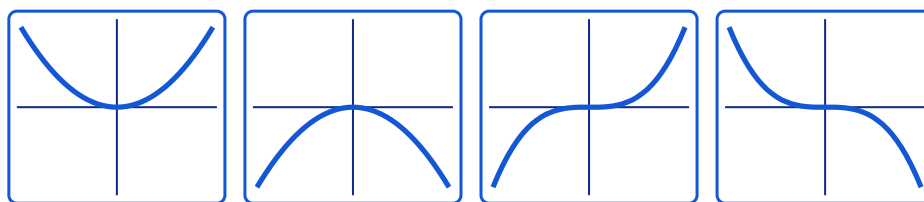
Grade 11 – Algebra 2 math · Lesson · Standards: F-IF.C.7c, A-APR.B.3, F-IF.B.4

Quick answer: A polynomial function's degree and leading coefficient set its end behavior, and the zeros of its factors are the x -intercepts. Odd multiplicity makes the graph cross the axis, and even multiplicity makes it touch and turn. A degree n polynomial has at most n real zeros and at most $n - 1$ turning points.

Standards: F-IF.C.7c, A-APR.B.3, F-IF.B.4 · **Level:** Grade 11 Algebra 2

How do the degree and leading coefficient control the graph?

A polynomial function in standard form is $f(x) = a_n x^n + \dots + a_1 x + a_0$. The highest power n is the **degree** and a_n is the **leading coefficient**. Together they fix the end behavior, because the leading term dominates when $|x|$ is large.



even degree, $a > 0$ even degree, $a < 0$ odd degree, $a > 0$ odd degree, $a < 0$

Degree parity picks the pattern; the sign of a picks the direction.

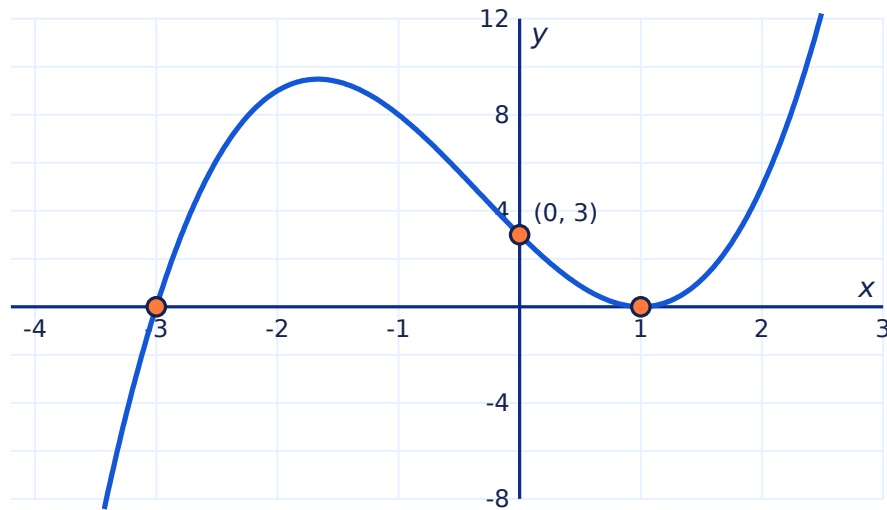
A polynomial of degree n has at most n real zeros and at most $n - 1$ turning points.

What do zeros and multiplicity tell you?

Zero and multiplicity

A **zero** of f is a number r with $f(r) = 0$, where the graph meets the x -axis. If $(x - r)^k$ is a factor, then r has **multiplicity** k . An odd multiplicity makes the graph cross the axis; an even multiplicity makes it touch and turn back. Higher multiplicity flattens the graph at the zero.

Factor	Zero	Multiplicity	Graph at the zero
$(x + 3)$	-3	1	Crosses straight through
$(x - 1)^2$	1	2	Touches and turns
$(x - 4)^3$	4	3	Crosses, flattened



$f(x) = (x + 3)(x - 1)^2$: the graph crosses at -3 and touches at 1 .

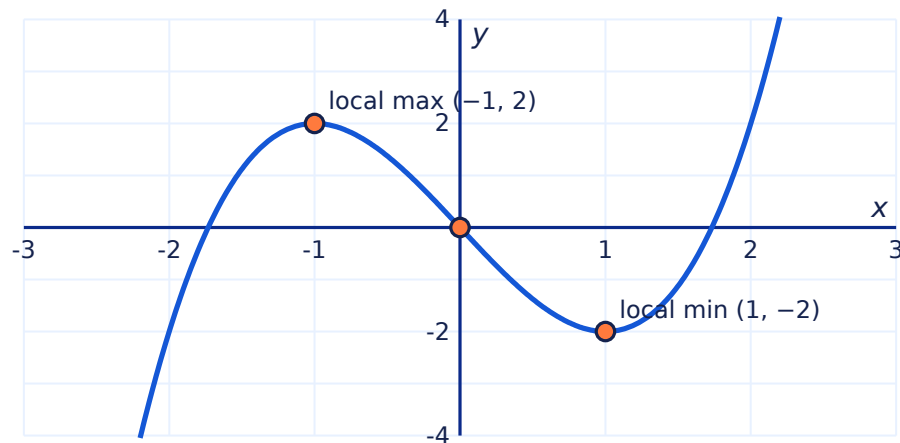
How do you sketch a polynomial from its factors?

Method: sketch a graph

1. Find the zeros and their multiplicities from the factored form.
2. Use the degree and leading coefficient to set the end behavior.
3. Find the y-intercept, $f(0)$.
4. Draw a smooth curve that crosses or touches at each zero as the multiplicity requires.

Worked example

For $f(x) = (x + 3)(x - 1)^2$, the degree is 3 and the leading coefficient is 1, so the left end falls and the right end rises. The y-intercept is $f(0) = 3 \cdot 1 = 3$.



A cubic has at most 2 turning points; this one has a local maximum and a local minimum.

Common mistake

A zero of even multiplicity does not change the sign of f . Do not draw the graph crossing at a

squared factor, and remember that turning points are not always zeros.

Mathbot's tip

Test one point between zeros to confirm the sign, then use the end behavior as a quick check on the first and last pieces.

Key points to remember

- The degree is the highest exponent; the leading coefficient is the coefficient of that term.
- Even degree: both ends point the same way. Odd degree: the ends point opposite ways.
- A positive leading coefficient makes the right end rise; a negative one makes it fall.
- Odd multiplicity: the graph crosses at the zero. Even multiplicity: it touches and turns.
- A degree n polynomial has at most n zeros and at most $n - 1$ turning points.

Polynomial Functions: Zeros, Ends, and Graphs: frequently asked questions

How do you find the degree of a polynomial?

Write the polynomial in standard form and find the highest exponent. If the polynomial is given as a product of factors, add the degrees of all the factors, counting repeated factors each time.

What does multiplicity mean for a graph?

Multiplicity is the number of times a factor appears. An odd multiplicity makes the graph cross the x -axis, while an even multiplicity makes it touch the axis and turn around. Larger multiplicities flatten the graph near the zero.

How many turning points can a polynomial have?

A polynomial of degree n has at most $n - 1$ turning points. A cubic can have 0 or 2, and a quartic can have 1 or 3.

Does every polynomial have real zeros?

No. An even-degree polynomial such as $x^4 + 1$ never reaches 0. An odd-degree polynomial always crosses the x -axis at least once because its ends point in opposite directions.

How can polynomials model real situations?

A volume that depends on a cut length, such as an open box made from a sheet, is a cubic function. The domain comes from the situation, and the graph helps find the maximum volume.