

Polynomial Functions: Zeros, Ends, and Graphs

Grade 11 – Algebra 2 math · Test Answer Key · Standards: F-IF.C.7c, A-APR.B.3, F-IF.B.4

Each answer lists the reasoning, the final result in bold, and the scoring guide. Give full credit for equivalent forms of a polynomial, and partial credit for a correct method with an arithmetic slip.

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Question 1 Degree and end behavior (2 points)

- a) Degree **5**, leading coefficient **−4**.
- b) Odd degree with a negative leading coefficient: **rises to the left and falls to the right**.

Scoring: 1 point each part.

Question 2 Zeros and multiplicity (2 points)

Zero 6: multiplicity 2, touches. Zero −1: multiplicity 1, crosses. Zero $-\frac{9}{2}$: multiplicity 3, crosses (flattened).

6 (2, touches); −1 (1, crosses); $-\frac{9}{2}$ (3, crosses)

Scoring: 1 point for the zeros, 1 point for multiplicities and behavior.

Question 3 Factor to find zeros (2 points)

$$x(x^2 - 7x + 10) = x(x - 2)(x - 5).$$

0, 2, 5

Scoring: 1 point for factoring, 1 point for the zeros.

Question 4 Build a polynomial (3 points)

$$f(x) = a(x + 4)(x - 1)(x - 3). \quad f(0) = a(4)(-1)(-3) = 12a = -24, \text{ so } a = -2.$$

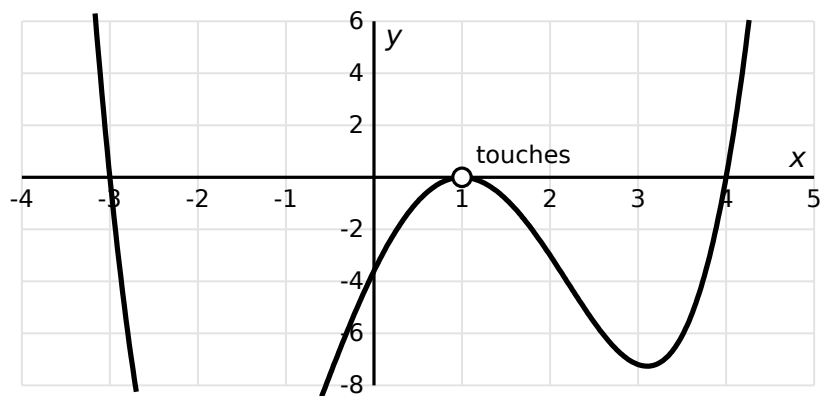
$$(x + 4)(x - 1)(x - 3) = x^3 - 13x + 12.$$

$$f(x) = -2x^3 + 26x - 24$$

Scoring: 1 point for the factored form, 1 for a, 1 for the expansion.

Question 5 Read a graph (3 points)

- a) Zeros $-3, 1, 4$. The graph touches at 1, so **1** has even multiplicity.
- b) Zeros $-3, 1$ (twice), 4 give degree **4**; both ends rise, so the leading coefficient is **positive**.
- c) $p(x) = a(x + 3)(x - 1)^2(x - 4)$ with $a > 0$.



The graph touches at 1.

Scoring: 1 point per part.

Question 6 Sign analysis (3 points)

Zeros: $-5, 2$ (even), 6 . Test points: $f(-6) = (-1)(64)(-12) > 0$; $f(0) = (5)(4)(-6) < 0$; $f(4) = (9)(4)(-2) < 0$; $f(7) = (12)(25)(1) > 0$.

$f(x) > 0$ for $x < -5$ and $x > 6$

Scoring: 1 point for the zeros and multiplicities, 1 for the sign tests, 1 for the answer.

Question 7 Modeling an open box (3 points)

- a) $V(x) = x(9 - 2x)(15 - 2x)$, domain $0 < x < 4.5$.
- b) $V(x) = 4x^3 - 48x^2 + 135x$.
- c) $V(1) = 1(7)(13) = 91$; $V(2) = 2(5)(11) = 110$; $V(3) = 3(3)(9) = 81$. **$x = 2$ gives the greatest volume, 110 cubic inches.**

Scoring: 1 point per part.

Question 8 Reasoning about graphs (2 points)

a) **No.** The ends of an odd-degree graph point in opposite directions, so the graph must cross the x-axis at least once.

b) **Yes.** A degree 5 polynomial has at most 4 turning points, and $x^5 - 5x^3 + 4x$ has exactly 4.

Scoring: 1 point each, including the explanation.