

Polynomial Functions: Zeros, Ends, and Graphs

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: F-IF.C.7c, A-APR.B.3, F-IF.B.4

Each solution identifies the key feature, such as multiplicity or sign, that decides the graph. Compare the sketch and the algebra: they must agree at every zero and at both ends.

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Problem 1 Degree and leading coefficient

Write each in standard form first.

- a) degree 4, leading coefficient 5: **quartic**.
- b) degree 3, leading coefficient -3 : **cubic**.
- c) degree 5, leading coefficient 6: **quintic**.
- d) degree 6, leading coefficient 4: **degree 6**.

Problem 2 End behavior

- a) Even degree, negative leading coefficient: **falls to the left and falls to the right**.
- b) Odd degree, positive leading coefficient: **falls to the left and rises to the right**.
- c) Odd degree, negative leading coefficient: **rises to the left and falls to the right**.
- d) Even degree, positive leading coefficient: **rises to the left and rises to the right**.

Problem 3 Zeros from factored form

- a) $x - 4 = 0$, $x + 7 = 0$, $2x - 5 = 0$: **4, -7 , $\frac{5}{2}$** .
- b) $x^2 = 0$ or $x + 3 = 0$: **0 and -3** .

Problem 4 Multiplicity

Zero -7 : multiplicity 3 (odd) crosses. Zero 3: multiplicity 2 (even) touches. Zero 8: multiplicity 1 crosses.

Degree = $3 + 2 + 1 = 6$.

-7 (3, crosses), 3 (2, touches), 8 (1, crosses); degree 6

Problem 5 Reading a polynomial graph

The graph crosses the x-axis at -2 , 1 , and 3 . It has a local maximum between -2 and 1 and a local minimum between 1 and 3 : 2 turning points.

With 3 zeros and 2 turning points, the least degree is 3.

Zeros -2 , 1 , 3 ; 2 turning points; degree 3

Problem 6 Factoring out a common factor

a) $x(x^2 - 9) = x(x - 3)(x + 3)$: **0, 3, -3.**

b) $x(x^2 + 4x - 21) = x(x + 7)(x - 3)$: **0, -7, 3.**

Problem 7 Quadratic form

Factor like a quadratic in x^2 : $(x^2 - 1)(x^2 - 16) = (x - 1)(x + 1)(x - 4)(x + 4)$.

$x = \pm 1$ and $x = \pm 4$

Problem 8 Factoring by grouping

Group: $x^2(x + 2) - 25(x + 2) = (x + 2)(x^2 - 25) = (x + 2)(x - 5)(x + 5)$.

-2, 5, -5

Problem 9 Writing a polynomial from its zeros

$f(x) = (x + 3)(x - 2)(x - 6)$. First, $(x + 3)(x - 2) = x^2 + x - 6$.

Then $(x^2 + x - 6)(x - 6) = x^3 - 6x^2 + x^2 - 6x - 6x + 36$.

$f(x) = x^3 - 5x^2 - 12x + 36$

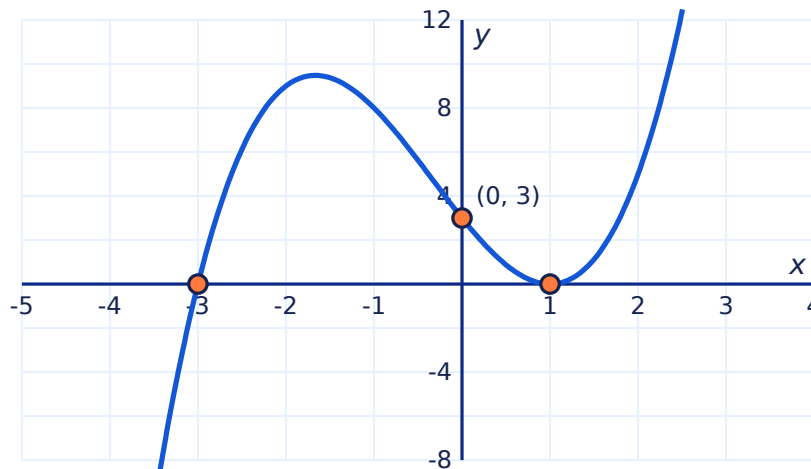
Problem 10 Using a point to find the leading coefficient

$f(x) = a(x + 1)^2(x - 4)$. Then $f(0) = a(1)(-4) = -4a = -12$, so $a = 3$.

$$f(x) = 3(x + 1)^2(x - 4)$$

Problem 11 Sketching from factored form

Degree 3, leading coefficient 1: falls left, rises right. Zeros: -3 (crosses), 1 (touches). y -intercept: $f(0) = 3$.



The completed sketch.

Crosses at -3 , touches at 1 , y -intercept 3

Problem 12 Sign of a polynomial

Test one point per interval. $f(-5) = (-1)(-6)(-10) = -60$. $f(0) = (4)(-1)(-5) = 20$. $f(2) = (6)(1)(-3) = -18$. $f(6) = (10)(5)(1) = 50$.

Negative, positive, negative, positive

Problem 13 Turning points and zeros

Turning points: at most $6 - 1 = 5$. Real zeros: at most 6. Yes: $f(x) = x^6 + 1$ is always at least 1, so it has no real zeros.

5 turning points; 6 zeros; yes, $x^6 + 1$

Problem 14 Where is the function positive?

Zeros: 0 (multiplicity 2), 3 (multiplicity 1), -6 (multiplicity 1). Degree 4, leading coefficient -1 : both ends fall.

$x^2 \geq 0$, so the sign comes from $-(x - 3)(x + 6)$, which is positive when $(x - 3)(x + 6) < 0$, that

is, $-6 < x < 3$. At $x = 0$ the function equals 0.

$f(x) > 0$ for $-6 < x < 0$ and $0 < x < 3$

Problem 15 Finding all zeros of a cubic

$f(-2) = 2(-8) - 3(4) - 11(-2) + 6 = -16 - 12 + 22 + 6 = 0$, so -2 is a zero and $(x + 2)$ is a factor.

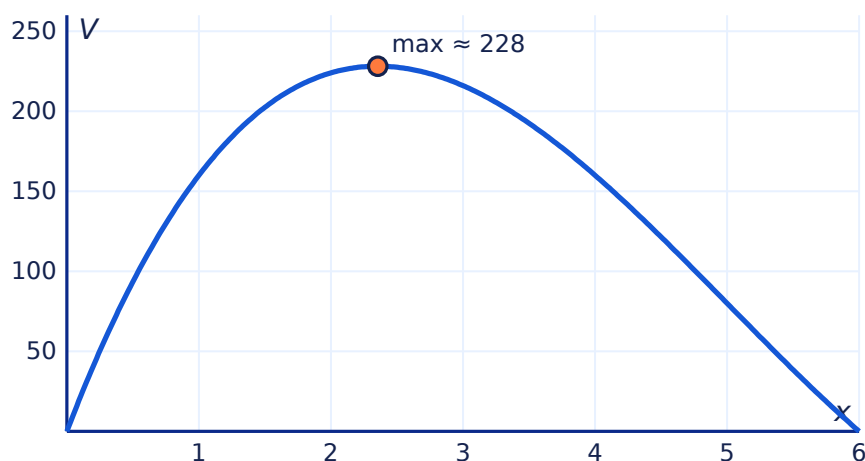
Dividing gives $f(x) = (x + 2)(2x^2 - 7x + 3) = (x + 2)(2x - 1)(x - 3)$.

Zeros $-2, \frac{1}{2}, 3$

Problem 16 An open box from a sheet

The box has height x , width $12 - 2x$, and length $18 - 2x$, so $V(x) = x(12 - 2x)(18 - 2x) = 4x^3 - 60x^2 + 216x$.

Each cut must leave positive width: $12 - 2x > 0$ and $x > 0$, so the domain is $0 < x < 6$.



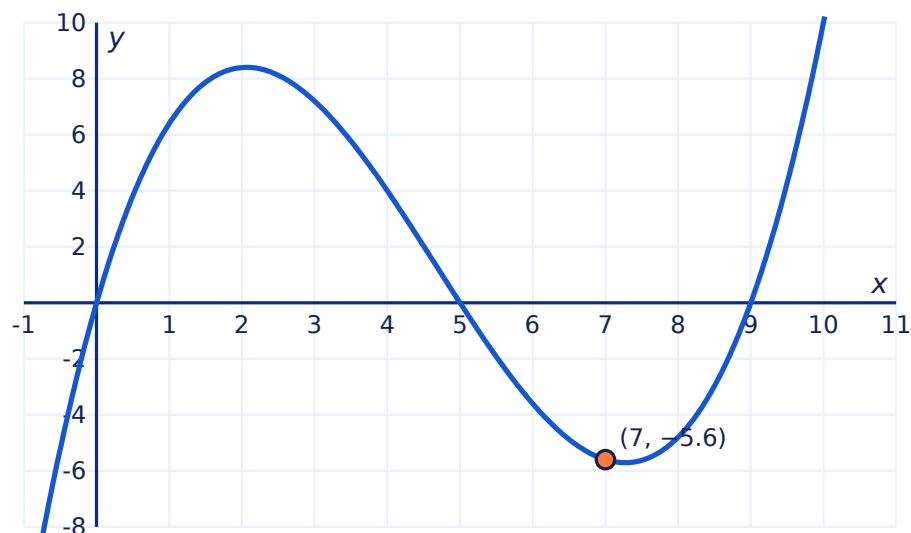
The maximum is near $x = 2.35$.

Domain $0 < x < 6$; greatest volume about 228 cubic inches at $x \approx 2.35$ inches

Problem 17 A tunnel in a roller coaster

Zeros: 0, 5, 9. With positive leading coefficient: positive for $0 < x < 5$, negative for $5 < x < 9$, positive for $x > 9$.

$$h(7) = 0.2(7)(2)(-2) = -5.6.$$



The point at $x = 7$.

Zeros 0, 5, 9; below ground for $5 < x < 9$; height -5.6 feet

Problem 18 A cubic through a point

$f(x) = a(x + 2)(x - 1)(x - 4)$. Then $f(2) = a(4)(1)(-2) = -8a = -16$, so $a = 2$.

$$2(x + 2)(x - 1)(x - 4) = 2(x^3 - 3x^2 - 6x + 8).$$

$$\mathbf{f(x) = 2(x + 2)(x - 1)(x - 4) = 2x^3 - 6x^2 - 12x + 16}$$

Problem 19 Identifying a function from its graph

Four zeros $(-3, -1, 1, 2)$, each crossing, so degree at least 4. Both ends fall, so the leading coefficient is negative.

$f(x) = a(x + 3)(x + 1)(x - 1)(x - 2)$. Then $f(0) = a(3)(1)(-1)(-2) = 6a = -3$, so $a = -\frac{1}{2}$.

Degree 4; negative leading coefficient; $f(x) = -\frac{1}{2}(x + 3)(x + 1)(x - 1)(x - 2)$

Problem 20 Analyzing a polynomial without graphing

Degree: $1 + 2 + 3 = 6$. Leading coefficient: 2.

Even degree and positive leading coefficient: both ends rise.

Zero 1 (multiplicity 1) crosses; zero -2 (multiplicity 2) touches; zero 5 (multiplicity 3) crosses and flattens.

$$g(0) = 2(-1)(4)(-125) = 1,000.$$

Degree 6, leading coefficient 2, both ends rise, y-intercept 1,000