

Probability Rules and Conditional Events

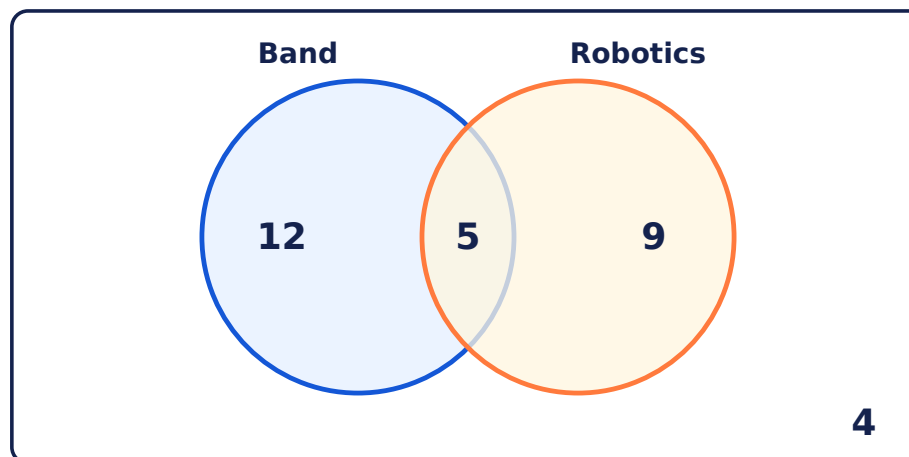
Grade 11 – Algebra 2 math · Lesson · Standards: S-CP.A.1, S-CP.A.2, S-CP.A.3, S-CP.A.4, S-CP.A.5, S-CP.B.6, S-CP.B.7

Quick answer: Probability rules combine events: the complement rule is $P(A') = 1 - P(A)$, and the addition rule is $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. Conditional probability is $P(A | B) = P(A \text{ and } B) \div P(B)$. Two events are independent when $P(A \text{ and } B) = P(A) \cdot P(B)$.

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What are events and how do you combine them?

Start from the sample space, the set of all possible outcomes. An event is any part of it. The complement of A , written A' , holds the outcomes not in A . The union $A \cup B$ means “ A or B ” and the intersection $A \cap B$ means “ A and B .”



30 students: 12 in band only, 5 in both, 9 in robotics only, 4 in neither.

Rules

- Complement: $P(A') = 1 - P(A)$
- Addition: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Multiplication: $P(A \cap B) = P(A) \cdot P(B | A)$

In the diagram, $P(\text{band or robotics}) = (12 + 5 + 9)/30 = 26/30$. The addition rule gives the same: $17/30 + 14/30 - 5/30 = 26/30$.

What does conditional probability mean?

Reading “A given B” means you look only at outcomes where B happened.

Conditional probability

$$P(A | B) = P(A \cap B) \div P(B)$$

	Coupon	No coupon	Total
Online	24	16	40
In store	18	32	50
Total	42	48	90

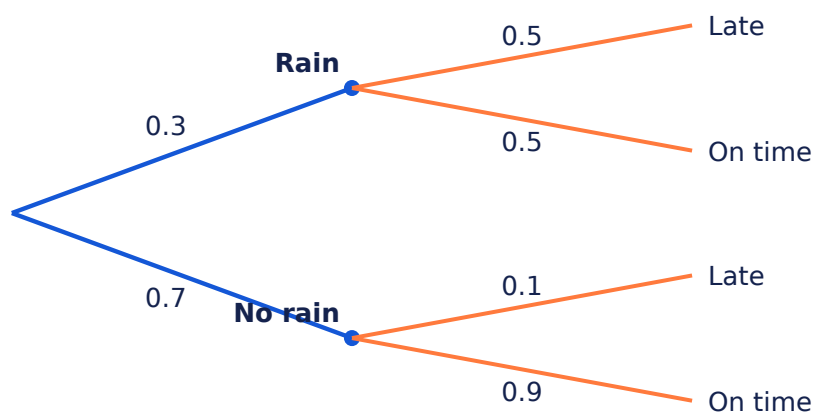
A two-way table of 90 shoppers.

Example

Among shoppers with a coupon, what fraction shopped online? Restrict to the Coupon column:
 $P(\text{Online} | \text{Coupon}) = 24/42 = \mathbf{4/7}$.

When are two events independent?

Two events are independent when learning that one happened leaves the chance of the other unchanged: $P(A | B) = P(A)$. That is the same as $P(A \cap B) = P(A) \cdot P(B)$. In the table, $P(\text{Online}) = 40/90 = 4/9$, but $P(\text{Online} | \text{Coupon}) = 4/7$, so the events are not independent.



A tree multiplies along branches: $P(\text{rain and late}) = 0.3 \times 0.5 = 0.15$.

Method: tree diagrams

1. Sketch one branch per first-stage result and label it with its chance.
2. Grow second-stage branches from each, labeled with conditional chances.

3. Multiply along each path. Add the paths that make up the event you want.

Common mistake

Treating mutually exclusive events as independent. If A and B cannot both happen, then $P(A \cap B) = 0$, and knowing A happened tells you B did not.

Mathbot's tip

For “given,” shrink the world to the “given” group first, then count inside it.

Key points to remember

- An event is a subset of the sample space; $A \cup B$ means A or B , $A \cap B$ means A and B .
- Addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- Conditional probability: $P(A | B) = P(A \cap B) \div P(B)$.
- Independent events satisfy $P(A \cap B) = P(A) \cdot P(B)$.
- Two-way tables and tree diagrams organize joint and conditional probabilities.

Probability Rules and Conditional Events: frequently asked questions

What is the difference between independent and mutually exclusive events?

Mutually exclusive events cannot happen together, so $P(A \text{ and } B) = 0$. Independent events can happen together and do not affect each other's probability. Two events with positive probabilities cannot be both.

Why do we subtract $P(A \text{ and } B)$ in the addition rule?

Outcomes in both A and B are counted once in $P(A)$ and again in $P(B)$. Subtracting the intersection once removes the double count so each outcome is counted exactly one time.

How do I read $P(A | B)$ from a two-way table?

Focus on the row or column of the given event B and use its total as the denominator. The numerator is the count of A within that group. Do not use the grand total.

How do I test whether two events are independent?

Compare $P(A \cap B)$ with $P(A) \cdot P(B)$, or compare $P(A | B)$ with $P(A)$. If the values are equal, the events are independent. Even small differences mean they are not.

When do I multiply and when do I add on a tree diagram?

Multiply along one path from left to right, because each branch is conditional on the previous one. Add the results of different paths when they all belong to the event you want.