

Radians and the Unit Circle Explained

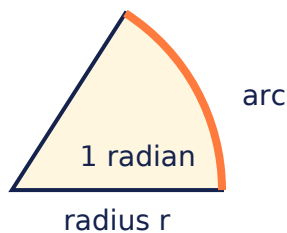
Grade 11 – Algebra 2 math · Lesson · Standards: F-TF.A.1, F-TF.A.2, F-TF.C.8

Quick answer: A radian is the angle that cuts off an arc equal to the radius, so 180° equals π radians. On the unit circle, the point at angle θ is $(\cos \theta, \sin \theta)$. The identity $\sin^2 \theta + \cos^2 \theta = 1$ gives one ratio from the other once the quadrant is known.

Standards: F-TF.A.1, F-TF.A.2, F-TF.C.8 · **Level:** Grade 11 Algebra 2

What is a radian?

A radian measures an angle by the arc it cuts off on a circle, compared with the radius. Because the full circle has arc $2\pi r$, a full turn is 2π radians.



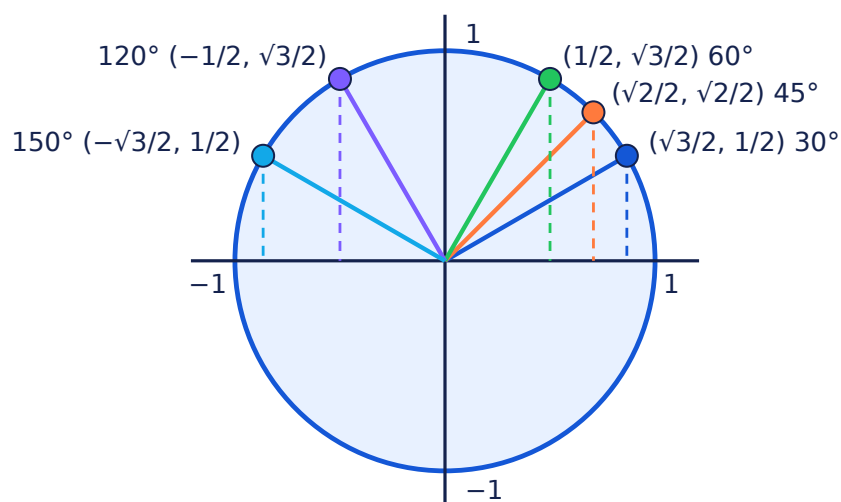
One radian is the angle whose arc is as long as the radius.

Conversions

π radians = 180° . Degrees to radians: multiply by $\pi/180$. Radians to degrees: multiply by $180/\pi$.
Arc length: $s = r\theta$ with θ in radians.

How does the unit circle define sine and cosine?

The unit circle has radius 1 and center $(0, 0)$. Rotate counterclockwise by θ from the positive x-axis. The endpoint is $(\cos \theta, \sin \theta)$, and $\tan \theta = \sin \theta \div \cos \theta$.

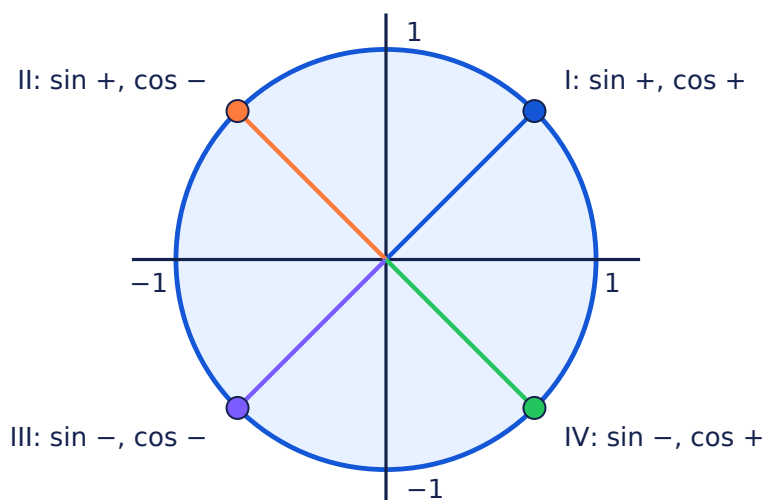


Each point on the unit circle is $(\cos \theta, \sin \theta)$.

Special-angle values

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\cos \theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\sin \theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1

Other angles use a reference angle, the acute angle to the x-axis, and the quadrant sign.



The signs of sine and cosine depend on the quadrant.

How do you find one ratio from another?

Pythagorean identity

$$\sin^2 \theta + \cos^2 \theta = 1$$

Method

1. Substitute the known ratio into the identity.
2. Solve for the other ratio squared, then take the square root.
3. Choose the sign from the quadrant.

Example

If $\sin \theta = 7/25$ and θ is in quadrant II, then $\cos^2 \theta = 1 - 49/625 = 576/625$. So $\cos \theta = -24/25$, since cosine is negative in quadrant II, and $\tan \theta = -7/24$.

Common mistake

Forgetting the sign after taking the square root. The identity gives only the size of the ratio.

Mathbot's tip

For 30° and 60° , remember that the smaller angle has the smaller sine: $\sin 30^\circ = 1/2$ and $\sin 60^\circ = \sqrt{3}/2$.

Key points to remember

- π radians = 180° , so a full turn is 2π radians.
- Arc length is $s = r\theta$ when θ is in radians.
- The terminal point of angle θ on the unit circle is $(\cos \theta, \sin \theta)$.
- $\sin^2 \theta + \cos^2 \theta = 1$ for every angle θ .
- Reference angles and quadrant signs give the exact value of any special angle.

Radians and the Unit Circle Explained: frequently asked questions

Why do mathematicians use radians instead of degrees?

Radians connect angle to arc length directly with $s = r\theta$, and they make formulas in calculus and trigonometry simpler. A degree is an arbitrary one 360th of a turn, while a radian comes from the circle itself.

How do I convert degrees to radians?

Multiply the number of degrees by $\pi/180$. For example, $150^\circ \times \pi/180 = 5\pi/6$. To go the other way multiply radians by $180/\pi$.

What is a reference angle?

It is the acute angle between the terminal side and the x-axis. Values of sine and cosine for any

angle equal those of the reference angle, up to a sign set by the quadrant.

How can I tell the sign of sine or cosine?

Sine is the y-coordinate, so it is positive above the x-axis (quadrants I and II). Cosine is the x-coordinate, so it is positive to the right of the y-axis (quadrants I and IV).

Why does $\sin^2 \theta + \cos^2 \theta = 1$?

The point $(\cos \theta, \sin \theta)$ lies on the unit circle, whose equation is $x^2 + y^2 = 1$. Substituting the coordinates gives the identity, which is the Pythagorean Theorem in disguise.