

Radians and the Unit Circle Explained

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: F-TF.A.1, F-TF.A.2, F-TF.C.8

Each solution names the reference angle, the quadrant and the sign. Compare these three pieces with your own work, because most mistakes are sign errors.

Standards: F-TF.A.1, F-TF.A.2, F-TF.C.8 · **Level:** Grade 11 Algebra 2

Problem 1 Degrees to radians

$150 \times \pi/180 = 150\pi/180 = 5\pi/6$. $5\pi/6$ radians.

Problem 2 Radians to degrees

$7\pi/4 \times 180/\pi = 7 \times 45 = 315$. 315° .

Problem 3 A radian measure without π

$4.2 \times 180/\pi = 240.6423$. $\approx 240.6^\circ$.

Problem 4 Special angle in quadrant I

The point at 60° is $(1/2, \sqrt{3}/2)$. $\cos \pi/3 = 1/2$ and $\sin \pi/3 = \sqrt{3}/2$.

Problem 5 Reference angle in quadrant II

The reference angle is $\pi - 5\pi/6 = \pi/6$, with point $(\sqrt{3}/2, 1/2)$. In quadrant II the x-coordinate is negative. $(-\sqrt{3}/2, 1/2)$.

Problem 6 Quadrant III values

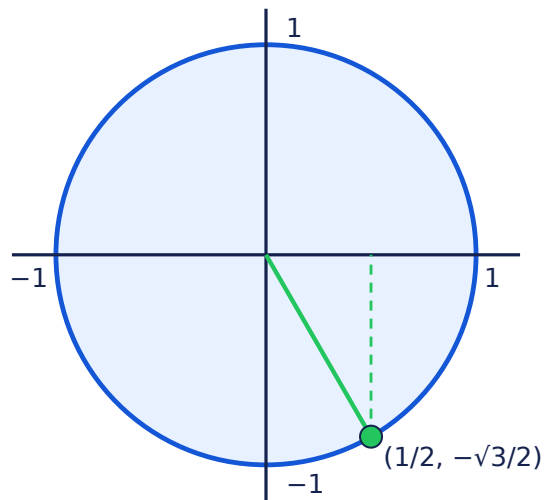
It lies in quadrant III. The reference angle is $4\pi/3 - \pi = \pi/3$. In quadrant III both coordinates are negative. $\sin = -\sqrt{3}/2$ and $\cos = -1/2$.

Problem 7 Tangent from coordinates

The point at $3\pi/4$ is $(-\sqrt{2}/2, \sqrt{2}/2)$. Then $\tan = (\sqrt{2}/2) \div (-\sqrt{2}/2)$. $\tan 3\pi/4 = -1$.

Problem 8 Read an angle on the unit circle

$5\pi/3$ is in quadrant IV with reference angle $2\pi - 5\pi/3 = \pi/3$. The point is $(\cos \pi/3, -\sin \pi/3)$. $(1/2, -\sqrt{3}/2)$.



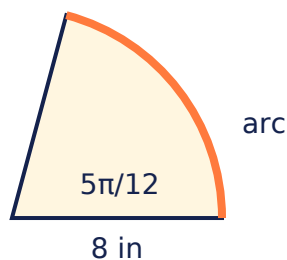
The point at $5\pi/3$ lies in quadrant IV.

Problem 9 Negative angle

A negative angle turns clockwise, ending at $(\sqrt{2}/2, -\sqrt{2}/2)$. **$\sin(-\pi/4) = -\sqrt{2}/2$ and $\cos(-\pi/4) = \sqrt{2}/2$.**

Problem 10 Arc length of a sector

$s = r\theta = 8 \cdot 5\pi/12 = 10\pi/3 \approx 10.47$. **$10\pi/3$ inches ≈ 10.47 in.**



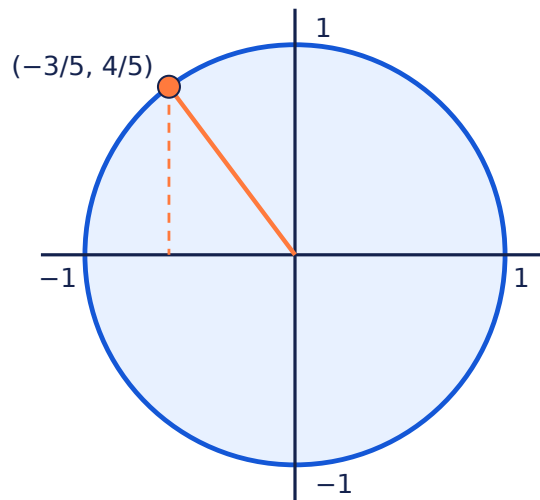
The arc is $10\pi/3 \approx 10.47$ in long.

Problem 11 Coterminal angle

$23\pi/4 - 4\pi = 23\pi/4 - 16\pi/4 = 7\pi/4$. **$\sin 7\pi/4 = -\sqrt{2}/2$. $7\pi/4$; sine = $-\sqrt{2}/2$.**

Problem 12 Point on the unit circle

$(-3/5)^2 + y^2 = 1$ gives $y^2 = 16/25$, and $y > 0$ in quadrant II. **$y = 4/5$, $\cos = -3/5$, $\sin = 4/5$, $\tan = -4/3$.**



$P = (-3/5, 4/5).$

Problem 13 Cosine from sine, quadrant II

$\sin^2 \theta = 1 - 25/169 = 144/169$, so $|\cos| = 12/13$. It is negative in quadrant II. **$\cos \theta = -12/13$ and $\tan \theta = -5/12$.**

Problem 14 Sine from cosine, quadrant IV

$\sin^2 \theta = 1 - 64/289 = 225/289$, so $|\sin| = 15/17$. It is negative in quadrant IV. **$\sin \theta = -15/17$ and $\tan \theta = -15/8$.**

Problem 15 Sine and cosine from tangent

In quadrant III both are negative, with $\sin \div \cos = 7/24$. Let $\sin = -7k$ and $\cos = -24k$. Then $49k^2 + 576k^2 = 1$, so $k = 1/25$. **$\sin \theta = -7/25$ and $\cos \theta = -24/25$.**

Problem 16 Ferris wheel position

The rider is at $(25 \cos 5\pi/4, 25 \sin 5\pi/4) = (-25\sqrt{2}/2, -25\sqrt{2}/2)$. Since $25\sqrt{2}/2 \approx 17.6777$: **about 17.68 ft to the left of and 17.68 ft below the center.**

Problem 17 Distance traveled by a clock hand

40 minutes is $40/60 = 2/3$ of a turn, so $\theta = 2/3 \cdot 2\pi = 4\pi/3$. $s = 9 \cdot 4\pi/3 = 12\pi \approx 37.70$. **$4\pi/3$ radians; ≈ 37.70 in.**

Problem 18 Show an identity

Expand: $\sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta$. Replace $\sin^2 \theta + \cos^2 \theta$ with 1. **$= 1 + 2 \sin \theta \cos \theta$.**

Problem 19 Two-step ratio problem

$\sin^2 \theta = 1 - 5/9 = 4/9$, so $\sin \theta = 2/3$ (positive in quadrant II). Then $\tan \theta = (2/3) \div (-\sqrt{5}/3) =$

$$-2/\sqrt{5} = -2\sqrt{5}/5. \sin \theta = 2/3 \text{ and } \tan \theta = -2\sqrt{5}/5.$$

Problem 20 Can both values be true?

Check the identity: $0.6^2 + 0.7^2 = 0.36 + 0.49 = 0.85$, which is not 1. **No such angle exists**, because $(0.7, 0.6)$ is not on the unit circle.